Quantum Computing Guide

for General Audiences to Technical Users in Denmark

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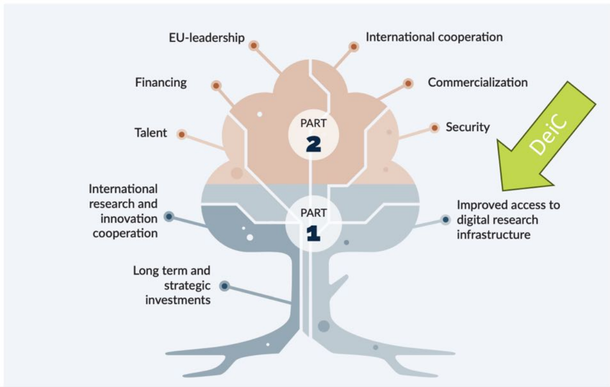
Preface

DeiC and the Danish National Quantum Strategy

As part of the implementation of the Danish National Quantum Strategy initiated in 2023 [1], DeiC has been allocated yearly funding to support various quantum initiatives. In particular, the Q-Access team as the author behind this manual, focuses on providing access to quantum computers and various test and technology platforms for quantum computing.



Useful for both non-technical audiences and quantum specialists

This overview is published by the Q-Access team - DeiC to provide a clear and smooth demonstration of the accessible quantum resource to all users in Denmark. In particular, it is designed to be useful to both non-technical audiences (policymakers, journalists, and industry leaders) and professional quantum users. It begins with a clear introduction to quantum computing principles, followed by descriptions of different quantum devices, real-world applications, and access methods.

Accessible quantum computing resources

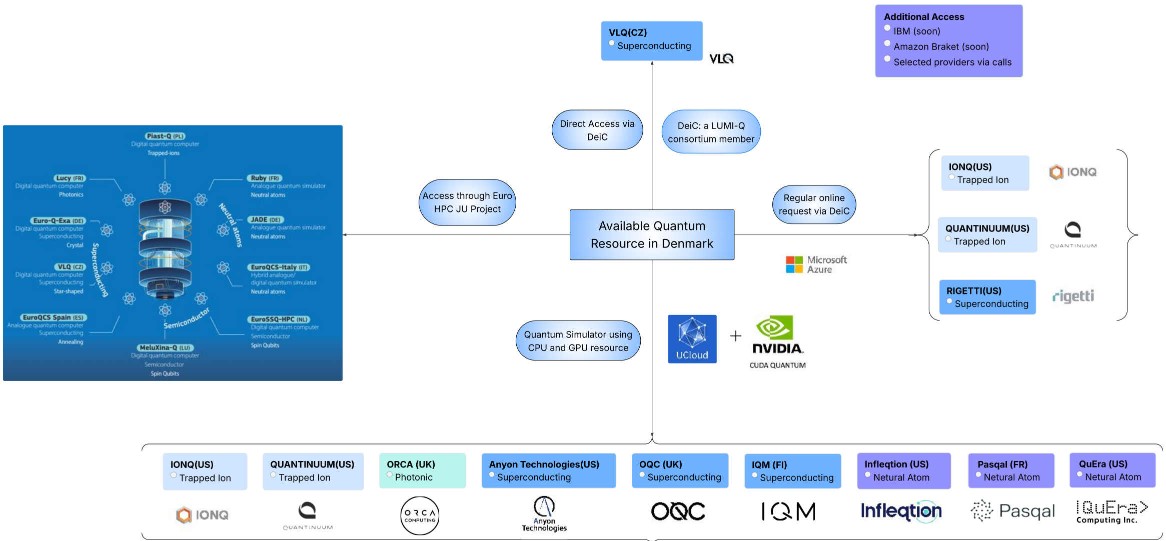


Figure 1: Quantum computing resources accessible to users in Denmark.

The mind map in Figure 1 illustrates the range of quantum computing resources currently accessible to Danish users under the support of DeiC. Researchers can apply for access to both sandbox environments and specific quantum hardware opportunities as well as consult our quantum experts.

Additionally, many quantum platforms are integrated to high-performance computing (HPC) facilities, enabling hybrid quantum–classical workflows for advanced tasks.

User's Manual: How to Read this Report

The aim of this section is to guide the reader on how to select suitable sections to read, thus allow each type of reader to focus on the most relevant parts.

Audience Groups

- **General audiences** (policy makers, journalists, industry newcomers): Focus on the introductory explanations and general applications.
- **Researchers and technical users:** Refer to the technical subsections including quantum algorithms and in-depth details.
- **Decision makers** (funding, infrastructure): See the comparative charts and access models that highlight advantages and limitations.

Reading Strategy

You do not need to read the report linearly; instead, you may navigate directly to sections of interest (key references are provided):

- Start with **Section 1** for a *historical background and conceptual introduction* based on:

  John Preskill: Quantum computing 40 years later

  IBM: What is quantum computing?

- Jump to **Section 2** for introduction to *quantum algorithms and their application regime*.

  Quantum Algorithm Zoo

- Jump to **Section 3–5** if you are mainly interested in *practical access*.

  DeiC Q-Access: Access to Quantum Computers

  VLQ: the quantum computer of the LUMI-Q consortium

  UCloud User Guide

  Google Quantum AI: Choosing hardware for your qsim simulation

- Jump to **Section 6-7** if you are mainly interested in *benchmarks, modalities and pricing models of different quantum computers*.

  Deep Lall et al: A Review and Collection of Metrics and Benchmarks for Quantum Computers: definitions, methodologies and software

- See **Section 8** for a description of the *EuroQHPC quantum-classical integration project*.

  The European High Performance Computing Joint Undertaking: Quantum Computers

- See **Section 9** for an extended description of the *quantum interface with AI*.

  Jacob Biamonte et al: Quantum Machine Learning

  Maria Schuld & Francesco Petruccione: Machine Learning with Quantum Computers

  LUMI: From binary computing to quantum AI

- See **Appendix** for additional useful services and initiatives from DeiC Quantum.

Contents

1. Motivation: Why Quantum Computing

Quantum mechanics provides the foundational laws of our universe. It governs phenomena across a wide range of regimes.

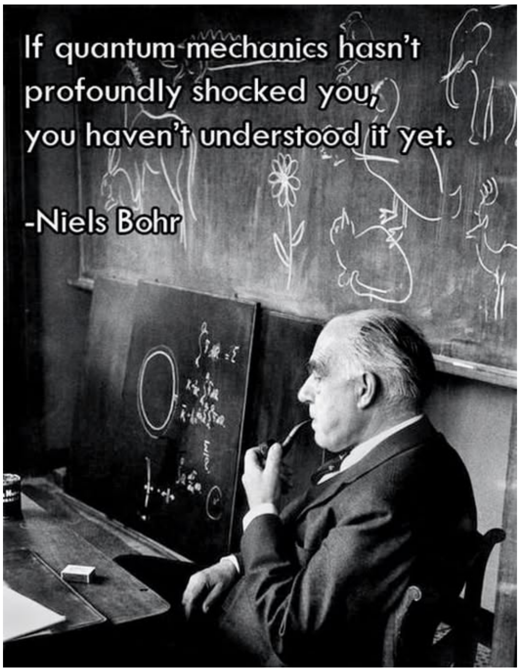
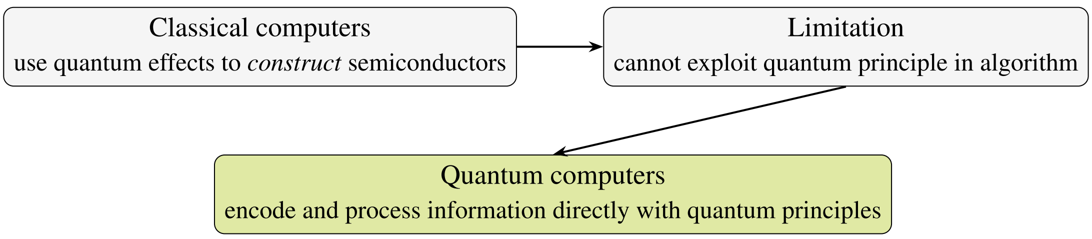


Figure 2: Quote from Bohr, who made foundational contributions to understand atomic structure and quantum theory

So far, we have achieved tremendous milestones of applications, for example (and numerous more subjects!):

- **Molecular properties** that influence biological cell functions;
- **Dynamical atoms** that determine the physical properties of materials;
- **Electron interactions** in semiconductors that enable modern chip manufacturing.

From classical computer to new quantum device



Goal: surpass classical HPC by harnessing superposition, entanglement, and interference.

Although the constructions of all classical computers are built on top of quantum-affected systems, the algorithms we design for classical computers do not change when quantum effects are used. In fact, they fail to take full advantage of the quantum-based principles during operation. This limitation has motivated researchers to propose an entirely new type of device, the *quantum computer*, with a completely novel way of encoding and processing information. Eventually, users can take advantage of quantum mechanics better or even fully to meet the computation demands that even the most high-performance computers cannot meet.

1.1. The theoretical foundation of quantum computing

Quantum computing is *simultaneously traditional and novel*! The rapid development of its foundation has profound historical roots back decades, yet its practical realization on physics devices remains remarkably recent.

The interplay between quantum mechanics and computer science has boosted significantly since the 1970s, both in academia and industry. Theoretical work on reversible computation and the thermodynamics of information processing by Bennett, Landauer, and Benioff [2–5] laid the key foundation for quantum device technology. Eventually in the 1980, two pioneering scientists, Feynman and Manin [6, 7] independently proposed a fundamentally new kind of device, the *quantum computer* - capable of exploiting uniquely quantum phenomena to perform computations in ways unreachable by classical machines.

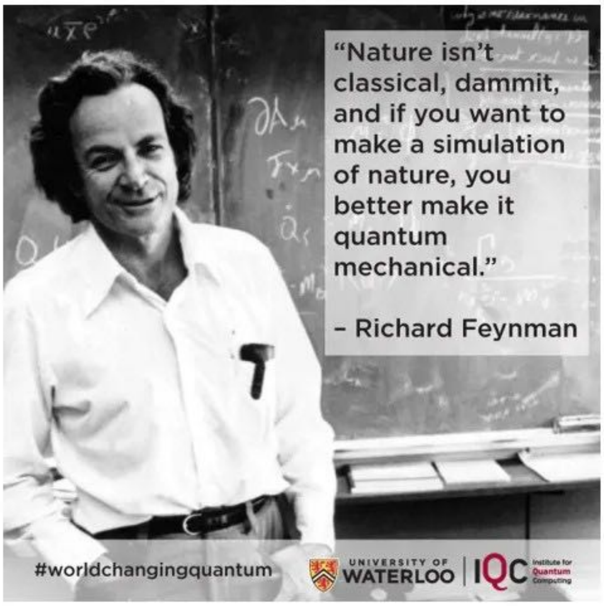


Figure 3: Quote from Richard Feynman (Source: U. of Waterloo)

The proposal was also aligned with the exponentially high computational demands to simulate complicated physical systems. Some might take a classical computer thousands of years to solve might be solved in a matter of minutes or hours by a quantum computer. However, realizing Feynman's vision is one of the great challenges of 21st century science and technology [8].

1.2. Current status of quantum devices construction

It was only in recent years that experimentalists successfully constructed small-to-medium-scale quantum computers. In 2019, Google AI and NASA announced that they had achieved quantum supremacy, marking the first realistic demonstration of a specific computation that classical computers are fundamentally unable to perform.

Today, the number of qubits has grown enormously, quantum computers with hundreds of qubits have now been implemented on various platforms [9–12], including superconductor, trapped ions, neutral atoms, quantum annealing, and photonics. In addition, researchers have achieved significant improvements in hardware and qubit control, as well as reducing error rates in quantum computers [9, 13, 14]. A complete overview of the development timeline for quantum computing is presented in Figure 4, highlighting the major milestones in the field.

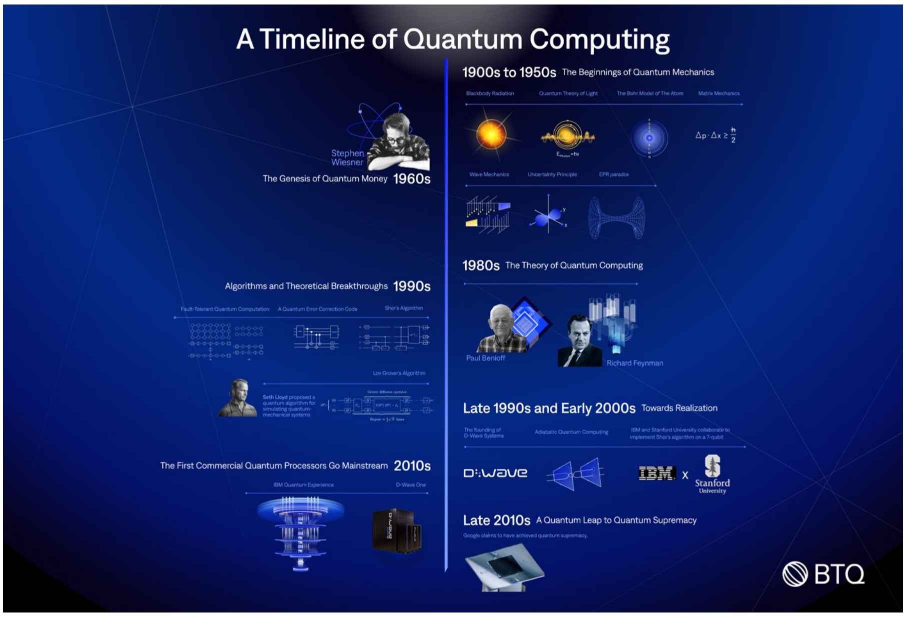


Figure 4: Quantum Computing: A Timeline (Source: BTQ company)

1.3. Classical computing versus quantum computing

Major scope of this review

- From this point onward, the content will focus on **gate-based quantum computing setups**, unless when explicitly referred to a specific non-gate-based platform.
- Non-gate-based models (e.g., quantum annealing or photonic sampling) are less developed than the gate-based models, and they employ different computational logistics and performance metrics than those introduced in the following content.
- Readers are therefore encouraged to make use of the **quantum consulting services provided by DeiC** or to contact hardware providers directly in order to obtain the most up-to-date performance evaluations and tailored guidance for their applications.

How classical computer works

From antiquated punch-card adders in Figure [5] to modern supercomputers in Figure [6], classical computers essentially function in the same way. These machines generally perform **calculations sequentially**, using binary bits of information, each represents either a 0 or 1 [15].

Figure 5: Hollerith's punch card counting machine in 1887

Figure 6: LUMI is a EuroHPC JU's flagship supercomputer and the 9th fastest globally (Top500 list published in June 2025).

How quantum computer works

Quantum computers can execute new algorithms that are deliberately designed using the principles of quantum physics. These algorithms consist of:

- Computational units: **Qubits**
- Operation units: **Quantum gates**

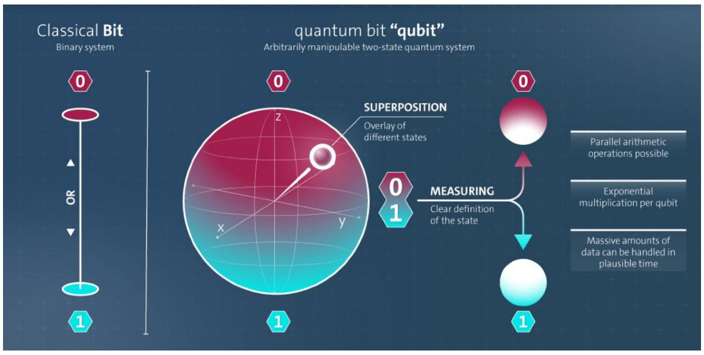


Figure 7: Qubit vs classical bit. Source: Volkswagen Aktiengesellschaft 2019

After all, we use *the quantum circuit* as a diagrammatic format of how a quantum algorithm is executed through the sequence of quantum operations (quantum gates).

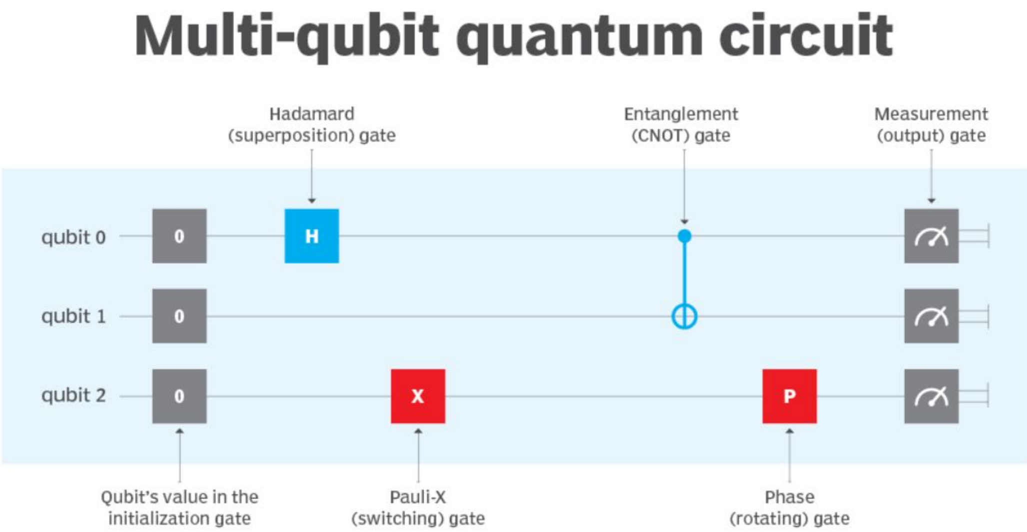


Figure 8: Components of a quantum circuit. Source: Leah Zitter.

Practical quantum gates

In practice, quantum hardware providers support their own sets of physically realizable gates. This means that when a quantum algorithm is compiled for execution, multi-qubit operations (such as general two- and three-qubit interactions) are decomposed into a series of specific one-qubit and two-qubit gates that are feasible on the given hardware platform. This decomposition is a critical step towards executable operations on real quantum processors.

Why quantum computers outperform classical devices?

Quantum computers outperform classical devices due to utilizing unique quantum phenomena, which completely lack classical equivalence. The two most known are:

Superposition to enhance information storage: A qubit itself is not very useful, but groups of qubits can store the quantum information in a state of superposition, which can create exponentially complex computational spaces for highly computationally demanding tasks.

Qubits	Memory Capacity	Comparison to Supercomputer
30	1.00 GB	$<$ Supercomputer
40	1.00 TB	$<$ Supercomputer
50	1.00 PB	$<$ Supercomputer
60	1.00 EB	$>$ Supercomputer
HPC	**Memory Capacity**	NA
El Capitan	5 PB	NA

Table 1: Memory required to store a full quantum state of n qubits (assuming 1 amplitude entry = 1 byte). 1 TB = 1,024 GB; 1 PB = 1,024 TB; 1 EB = 1,024 PB. In comparison, the most powerful supercomputer *El Capitan* has $\sim$5 PB of memory [23].

Entanglement to correlate the system: entangled systems are so intrinsically linked, such that when quantum processors measure a single entangled qubit, they can immediately determine (partial) information about other qubits.

Visualizing quantum entanglement [24]

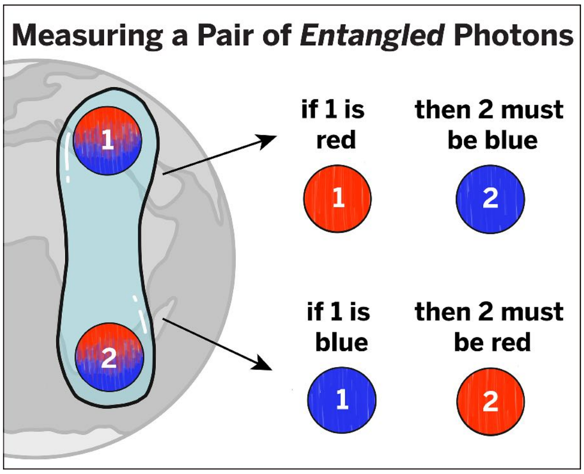


Figure 9: Entanglement governs how color of one partner relate to measurements of another.

Let us consider two particles of light, which scientists call photons. Now two photons are entangled, such that each photon has a chance of being either blue or red, but, once measured, they are always different colors. If the first photon is observed blue, we would immediately know that the second photon is red without eye perceiving. And vice versa.

Benefits of entangled system

1. Quantum parallelism, making it possible to perform computations on all correlated states simultaneously. Provides Quantum Advantage in Algorithms

2. Execution of quantum algorithms (e.g., Shor's factoring, Grover's search, quantum Fourier transform), as they require entanglement to create and manipulate highly non-classical correlations, which lead to exponential or quadratic speed-ups over classical algorithms.

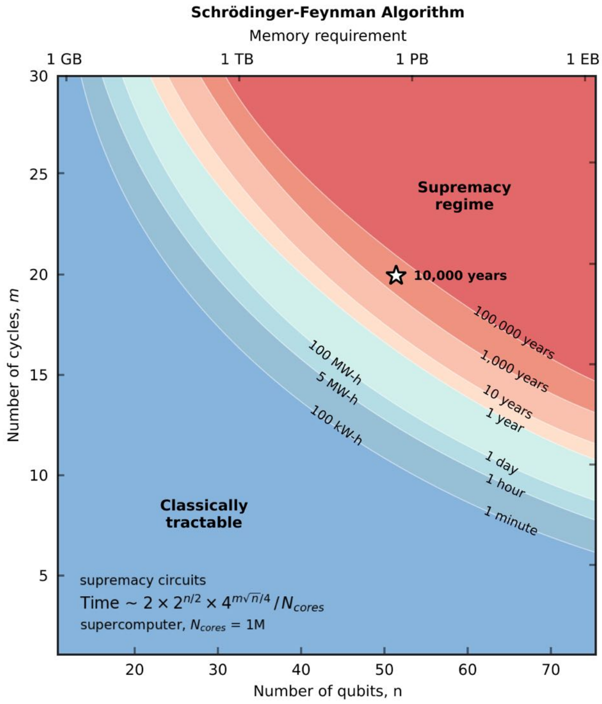


Figure 10: Estimate of the equivalent classical computation time for quantum supremacy circuits of the Schrödinger-Feynman algorithm. The star shows the estimated computation time for the largest experimental circuits. Source: Google AI Quantum, 2019

Classical vs. Quantum: Application Scenario

Classical computers are expected to remain the best solution for general everyday tasks. However, when encountering certain highly complex problems, classical devices become expensive and time-consuming to process, even with no solutions on any practical scale.

1.4. Two quantum computing developing stages: NISQ vs. FTQC

NISQ (Noisy Intermediate-Scale Quantum) and FTQC (Fault-Tolerant Quantum Computing) refer two distinct phases in the development of quantum computing.

NISQ (current era)

NISQ devices represent the current frontier of quantum computing technology. They are characterized by noisy qubits and the absence of full error correction. Typically the devices operate at a scale of tens to a few hundred qubits, making them suitable for specific tasks but not for executing large-scale algorithms such as HHL.

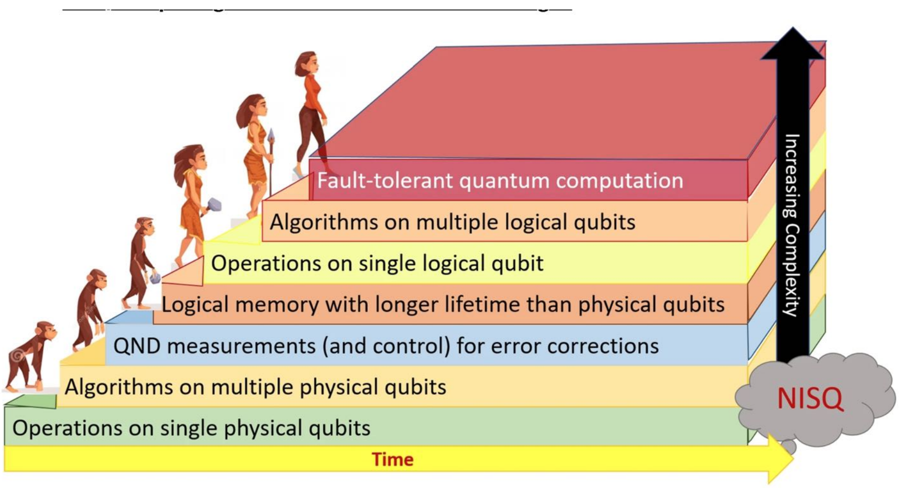


Figure 11: Different stages of quantum computing development [16].

FTQC (future)

While today's quantum devices already reach qubit counts in the hundreds to satisfy the need for certain real-world applications, error rates remain too high to make the results of enough computation useful. Future Fault-Tolerant Quantum Computers (FTQC) will enable robust error correction to realize reliable performance of complex algorithms at large scale.

1.5. Lowering the energy footprint : Another benefit

High demanding classical data centers and supercomputers draw substantial power. For example, the Frontier supercomputer, hosted at the DoE Oak Ridge Laboratory in the U.S., uses 504 MWh on average daily, summing up the energy consumed by around 17 thousand average homes in the U.S. daily. However, today's quantum processors typically operate at *orders of magnitude* lower electricity use, which is far below flagship HPC systems. This gap suggests a potential *energy* advantage. [1]

System	Power(kW)	Cooling consumption
Frontier supercomputer	21,000	33% – 40%
Superconducting	25 - 140	64%
Trapped Ions	2	15%
Neutral atom	7-20	$\sim$ 50%
Photons	4	75%

Table 2: Illustrative electricity use. Source: Olivier Ezratty CC, 2023.

[1] Summarized from Pasqal, *Quantum Computing to Greener Calculations* (Oct. 5, 2023).

An illustrative example from Quantinuum

A concrete quantitative comparison is provided by Quantinuum [17], who estimate the relative energy cost of evaluating the Jones polynomial using two paradigms. In this demonstration, a classical tensor-network algorithm (MPO-PROJ) is assumed to run on the most energy efficient supercomputer: Jülich JEDI [18] with a peak efficiency of 2.6×10^{17} FLOPS/kWh. The quantum circuit implementation (CFEV) is implemented on the Quantinuum H2 trapped-ion system [19]. Under these conditions, the quantum algorithm achieves a substantial reduction in total energy consumed per problem instance, since it requires exponentially fewer computational steps for equivalent accuracy at large scales.

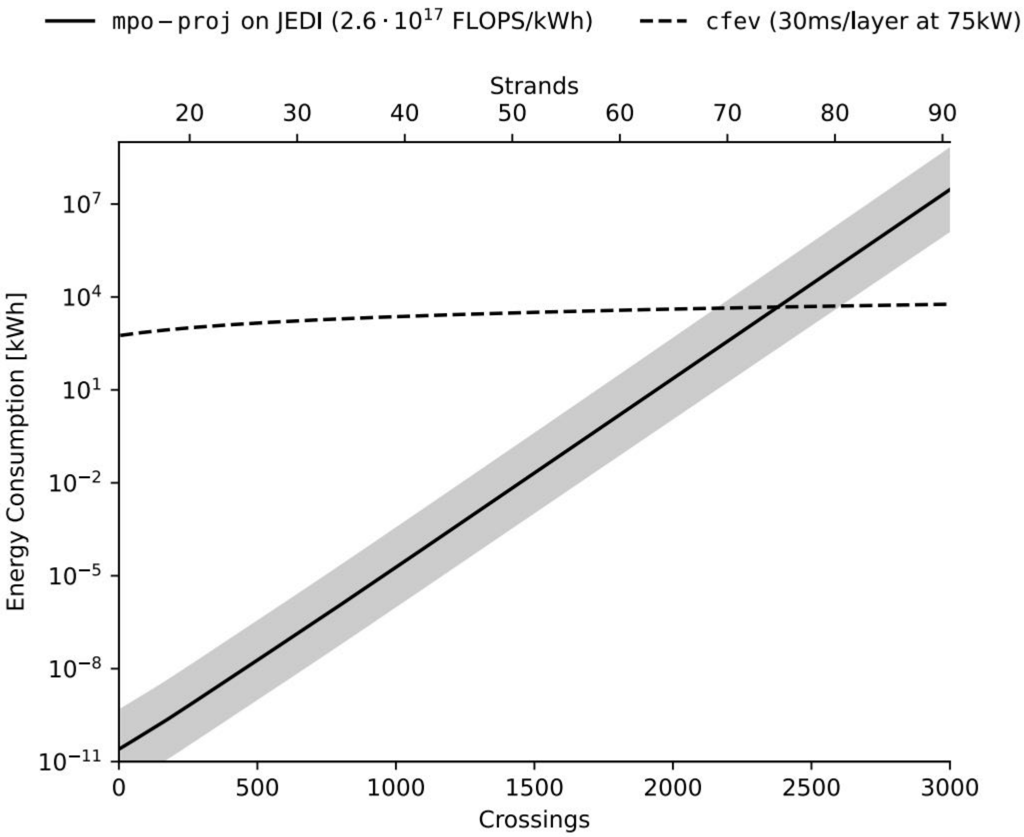


Figure 12: Comparison of estimated energy consumption of Quantinuum's H2 quantum computer and the most efficient classical computing cluster as of November 2024: the JEDI system at the J¨ulich Supercomputer Center. It is expected that cfev used by Quantinuum becomes more energy efficient than mpo-proj at large scales ($c \geq 2400$).

This example concretely illustrates how quantum computing can lead to energy savings on specific tasks, even before large-scale fault-tolerant operation is achieved.

The long term anticipation

The long-term quantum technologies energy usage is unknown due to the considerable energy cost of scaling quantum computing power and error correction consumptions.

The figure below, provided by Alexia Auffèves and Olivier Ezratty, 2022, shows a general comparison of energy consumption anticipations in terms of the size of a classical intractable problem. There, we can see that even with the introduction of error corrections, we will still have an interesting energy advantage for quantum computers.

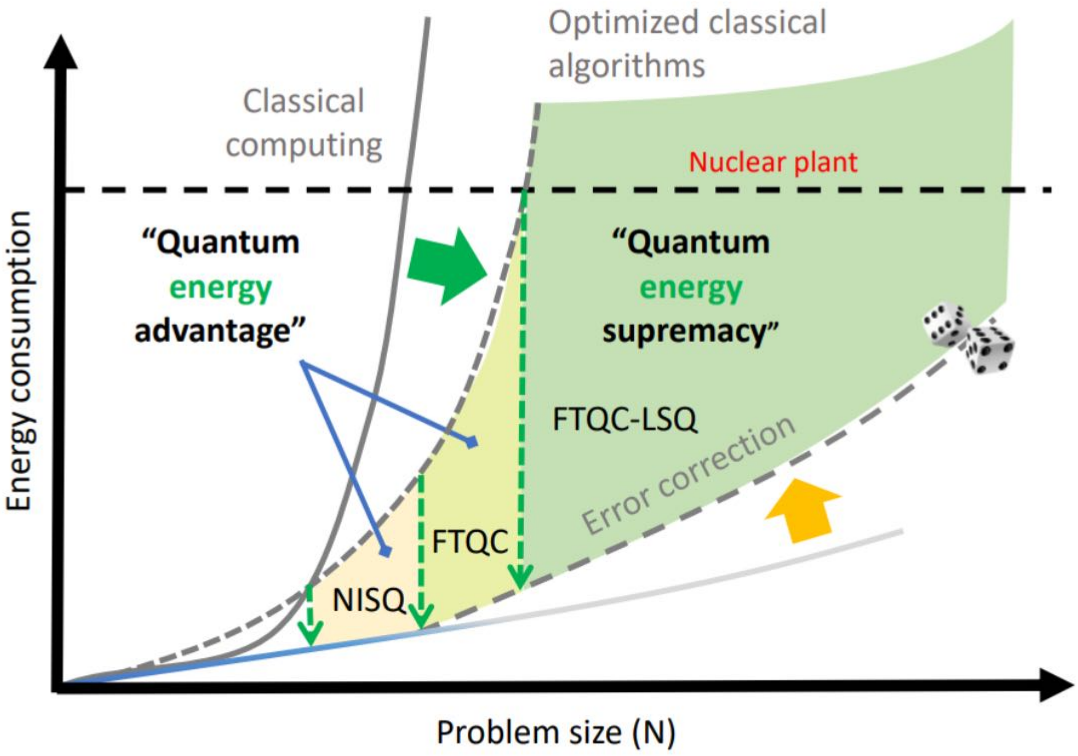


Image by Alexia Auffèves and Olivier Ezratty, CC 2022.

There remain open questions about whether the potential energy advantage of quantum computing will persist across all algorithms and applications. Supercomputers remain indispensable for advancing science, enabling high-precision simulations and solutions to global challenges such as climate change. Yet, to truly combat climate change, providers of both classical and quantum computing must continue reducing their energy footprint, while society as a whole strives toward the broader goals of sustainable development.

2. Introduction to quantum algorithms and applications

In 1994, mathematician Peter Shor published one of the first practical real-world applications for quantum computing [20]. It demonstrated the potential of quantum systems to solve problems not just for cybersecurity, but also for many other fields.

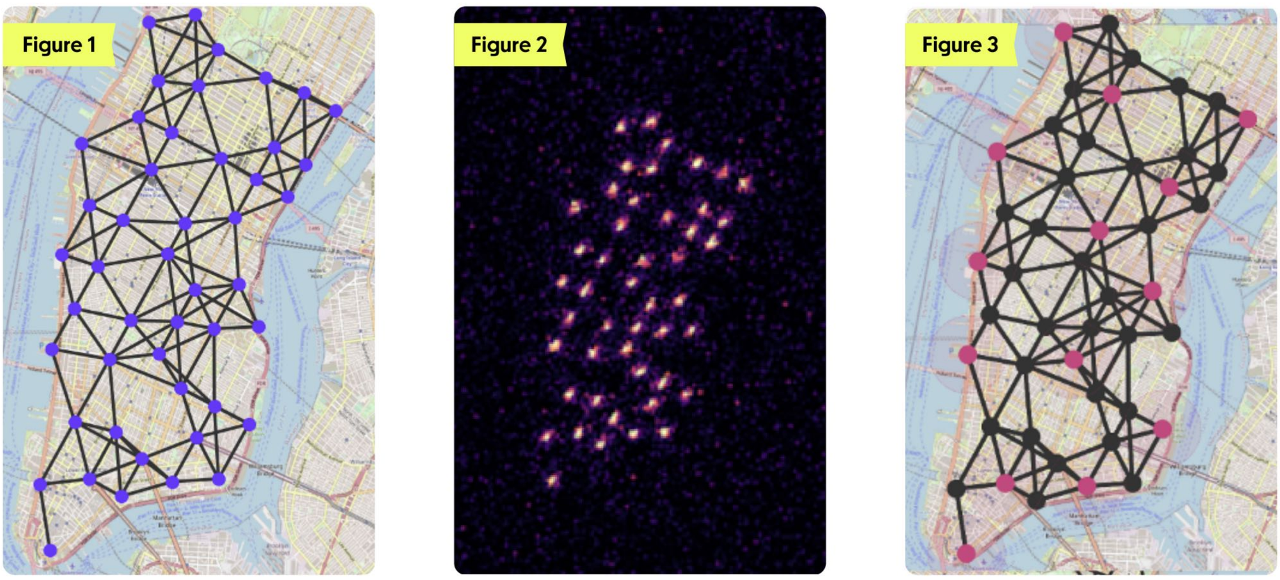
However, Shor's algorithm can be difficult to comprehend for a general audience without a background in quantum computing. Therefore, we begin this section with a simple illustrative example that avoids complicated mathematical formulas, while still demonstrating how quantum computers can address practical problems. This toy model is adapted from QuEra's quantum use case material.

2.1. A Toy Model - Store Selection in Manhattan Market

The problem concerns a market decision maker seeking to enter a new region by opening multiple coffee shops in Manhattan. As a first step, several viable shop locations are identified, represented as blue dots in the left of Fig. 13. However, not all locations can be chosen: some are too close to one another and would overlap in customer coverage. This introduces an *independent set* constraint in graph theory, such that optimal solutions cannot place two coffee shops within a given distance.

Some background knowledge of graph theory

In graph theory, the basic elements are *nodes* (also called vertices) and *edges*. In this coffee shop selection problem, each node corresponds to a candidate shop location, and each edge represents a "too-close" relationship, indicating that the two connected locations cannot both host a shop. The independent set constraint is therefore equivalent to select the largest possible set of nodes (shop locations), such that no two nodes connected by an edge are chosen simultaneously.



An example set of store locations. Each vertex is a potential location; each edge is an independent set restriction that no adjacent locations can both have stores simultaneously.

A photograph of individual atoms positioned in Aquila's atomic array encoding the store placement problem. In this example, each atom corresponds to a potential store location.

An example maximum independent set solution shown in red vertices.

Figure 13: A quantum application to optimize store locations. Source: Jonathan Wurtz, QuEra.

Encoding the data into quantum hardware and Solve the problem

In neutral atom hardware, e.g., the Rydberg atoms [21], each qubit is encoded in the electronic state of a neutral atom. The graph problem can be directly mapped to the arrays of the atoms in an analogous way, as shown in the middle of Fig. 13. Laser pulses excite the atoms, spreading entanglement across the system. By tuning the laser amplitude and phase over time, the final state encodes the solution, which is read out through the measured presence or absence of atoms in the array.

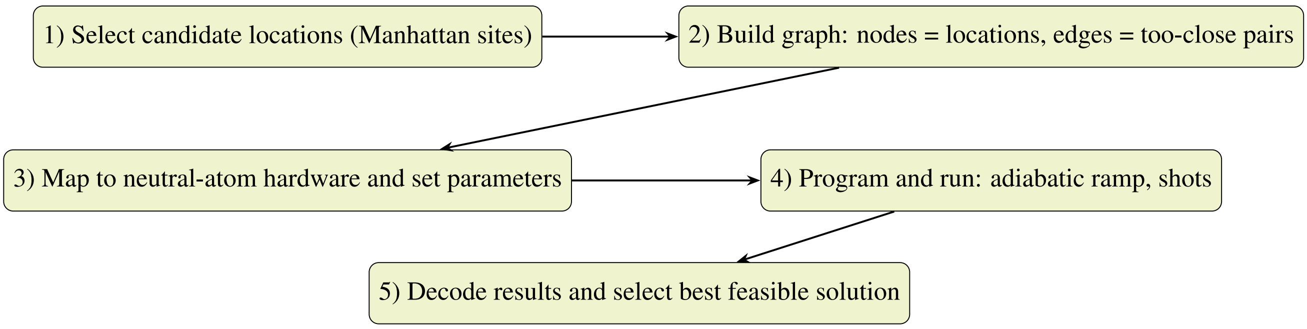


Figure 14: Procedure for optimize shop location selections: candidate locations and graph construction at the top; embedding, execution, and decoding at the bottom.

The optimized store placement is shown on the right of Fig. 15. In addition, a comparison of classical vs. quantum performance is shown in Fig. 13, where the quantum algorithm outperforms a simple greedy classical heuristic. Meanwhile, it should be noted that more sophisticated classical algorithms exist and currently exceed the performance of the quantum optimizer.

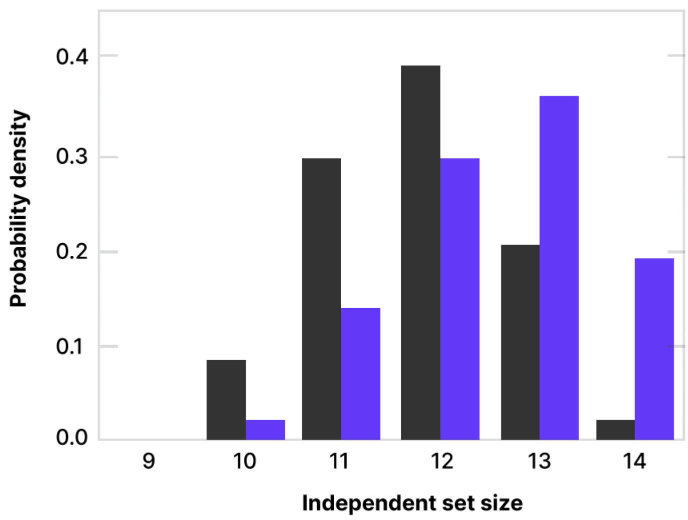


Figure 15: Performance of Aquila solving the MIS problem (purple), in comparison to a simple greedy heuristic classical optimizer (black). The quantum (classical) algorithm has an 18.8% (2.7%) chance of finding the MIS.

Generalized application beyond this toy model

Atoms in this type of hardware configuration can be arranged in arbitrary configurations in a 2d plane, so local interactions are naturally encoded. Therefore, the same approach extends beyond this retail example to other domains, such as logistics optimization, or even to modeling molecular structures for drug discovery.

2.2. Quantum computing use cases

Today, quantum technology is shifting from research-driven exploration to industry-driven applications across sectors such as chemicals, life sciences, logistics, automotive, financial services, and cybersecurity. Increasingly, stakeholders are investigating how quantum computers could address critical challenges and unlock novel solutions within their respective domains.

	Molecular simulation	Quantum optimization	Quantum Monte Carlo	Machine learning	HHL	Decryption [1]
Life sciences	Calculating a drug's binding affinity	Optimizing the location of clinical trial sites	Predicting the spread of disease in epidemics	Improving image classification in diagnostics	Modelling forces for protein - folding simulations	Protecting patient data privacy
Chemicals	Simulating the reaction pathway in synthesis	Optimizing the production process of chemicals	Simulating meso - scale reactor processes	Predicting the properties of new chemicals	Solving fluid dynamics in reaction vessels	Protecting data related to IP and trade secrets
Energy	Designing new materials for carbon capture	Optimizing power dispatching in an electric grid	Forecasting energy prices in the market	Predicting energy production from weather patterns	Solving DC power flow calculations in electrical grids	Protecting access to data on grid infrastructure
Telecom	Designing new semiconductor materials	Optimizing antenna placement	Stress - testing network resilience	Improving customer segmentation	Solving EM - field calculations in antenna design	Protecting the data exchanged over a network
Advanced manufacturing industries	Designing new batteries for electric vehicles	Optimizing the step sequence in car production	Improving the resilience of the supply chain	Improving fault detection in chip manufacturing	Solving aerodynamics simulations	Protecting communication connections
Logistics	N/A	Optimizing the route of a delivery service	Stress - testing logistic schedules for disruptions	Predicting maintenance needs in a fleet	Improving inventory management	Protecting personalized customer data
Finance	N/A	Optimizing the value of an asset portfolio	Modelling credit value at risk in capital allocation	Improving the detection of fraud in transactions	Estimating risk for the future value of an asset	Protecting customer transaction data

Figure 16: Overview of major quantum use cases, by industry sector and type of quantum algorithm. Source: ALICE & BOB.

Ongoing Development of Danish quantum ecosystem

Meanwhile, numerous initiatives are currently igniting in the Danish quantum ecosystem. The Danish quantum community, in collaboration with the Danish Business Authority, has published a report to include a collection of 16 emerging real-world use cases. This report illustrates the ongoing development of Denmark's quantum landscape and highlights how organizations in various sectors are engaging with diverse quantum technologies, taking their first steps toward practical applications [22].

Figure 17: 16 Danish Quantum Use Cases: Praktisk indsigt i brug af kvanteteknologi i industrierne. Source: Danish quantum community & Danish business authority.

3. User groups - from academia to industry

To help users in Denmark quickly understand their access rights, we summarize in the following table which services and grant opportunities are available for academic researchers and industry users. This overview aims to guide stakeholders to the appropriate channels for applying funds and accessing quantum computing resources and services.

Service and Resource	Academia	Industry
Microsoft Azure Quantum	✓	–
Specific Quantum Hardware Access	✓	?
PhD and Postdoc Scholarship	✓	✓
EuroHPC JU Call (Quantum)	✓	?
Quantum Consulting Service	✓	✓
Access to LUMI Q	✓	?

Table 3: Overview of services and grants for academia and industry. *Limited access refers to specific business-support schemes (e.g., business postdoc grants).

4. Quantum resources available through DeiC

Denmark is actively engaged in several national and European initiatives to provide users with access to cutting-edge quantum computing resources, as shown in Figure 18.

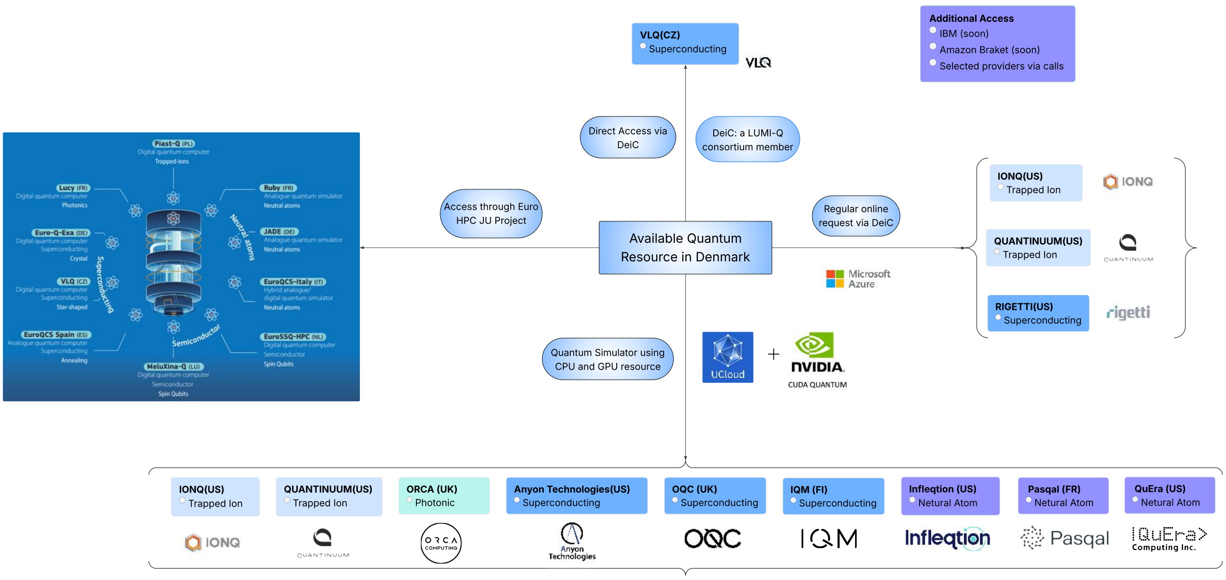


Figure 18: Quantum Resources accessible to users in denmark

Further details on properties, pricing models, and other relevant information for each quantum computer are provided in Section 7.

Scope of the Current and Following Section

The following content in this section covers resources directly accessible through DeiC. Users seeking additional access are encouraged to consult Section 5 (UCloud allocations for Danish universities) and Section 8 (partnerships under the EuroHPC JU project).

4.1. Sandbox Access - Microsoft Azure Platform

DeiC provides access to the Microsoft Azure Quantum cloud service (via the European OCRE framework) to Danish academia. See below for more information on eligibility.

Who is eligible to apply for quantum access? –

Danish universities can apply.

For universities, the following conditions apply:

Professors, associate professors, and assistant professors can apply for themselves and also on behalf of post docs, PhDs and master's thesis students.

To promote the use of quantum resources in all scientific fields, DeiC has pre-reserved 10% of the offered resources for possible applications from the area of social sciences and humanities (SSH). If no qualified applications are received, the resources will be transferred to the overall pool.

Figure 19: Eligibility for applicants

Through Microsoft Azure Quantum, users have access to a diverse portfolio of quantum simulators and quantum computers from Quantinuum, IonQ and Rigetti. This access is intended for testing, so users are encouraged to start with various free simulators before moving on to actual hardware tests. Fill out a form here to request access to Microsoft Azure Quantum.

Figure 20: Q Access via Microsoft Azure

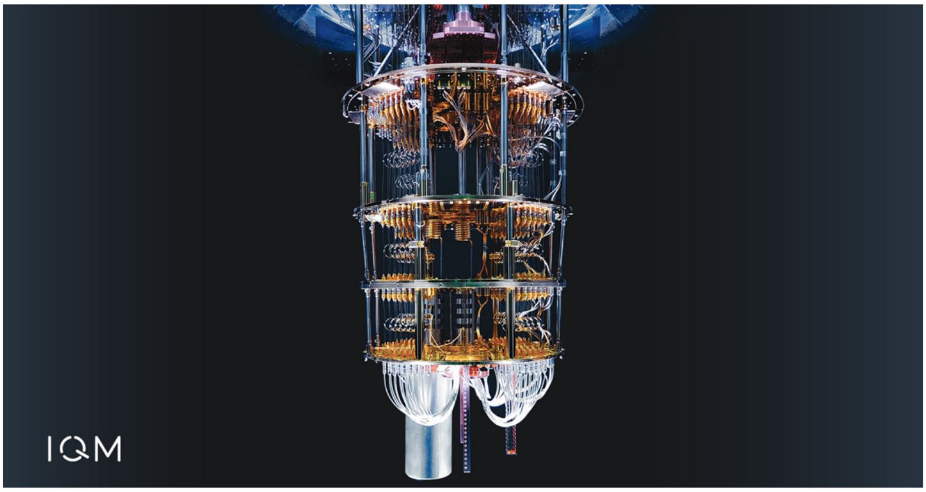
Remark Upon certain dates (currently 30/08/2025?), unused resources will be returned to the pool, and a new usage period will begin. Users will be asked to fill out a short report on their usage, which will be required for future allocations of resources.

4.2. LUMI-Q Consortium - participated by DeiC

DeiC participates in the LUMI-Q consortium under EuroHPC JU project to enhance the European quantum computing infrastructure [28]. The contract of acquisition was signed between the European High Performance Computing Joint Undertaking (EuroHPC JU) and IQM Quantum Computers, the company selected to supply the unique technology.

Europe takes a quantum leap: LUMI-Q consortium signs contract to establish quantum computer in the Czech Republic

26.9.2024



4.2.1. Properties of VLQ

VLQ is a EuroHPC quantum computer based on superconducting qubits, which is located in Ostrava, Czechia. The star-shaped qubit arrangement provides a one-to-all-qubit connectivity, thus improving the fidelity and coherence of quantum operations. The system will enable European end-users to actively explore applications and algorithms tailored for the novel star qubit topology with pulse level control, such as e.g. hardware-efficient quantum error correction (QEC) schemes or Quantum Fourier Transform (QFT), showing exponential speed-ups compared to purely classical processing.

4.2.2. Access to the upcoming LUMI-Q VLQ quantum computer

DeiC participates in the LUMI-Q consortium under EuroHPC JU and will also provide access to the upcoming LUMI-Q VLQ quantum computer for Danish users.

Meanwhile, DeiC will provide onboarding sessions and consulting to help Danish academia and industry take full advantage of VLQ (in addition to what will be offered by IQM and IT4I). This close HPC integration and pulse-level access will open new possibilities for applications and diversify DeiC's Q-Access offering.

Feature	Description
Modality	Superconducting qubits (24 total) in a star-shaped topology.
Gate Fidelity	>99%
Coherence	35 µs ($T_1 = 35$ µs).
Hosting Site	IT4Innovations, National Supercomputing Centre (IT4I) - Czechia
Hosting Supercomputer	EuroHPC KAROLINA (12.9 PetaFlops)
Programming	Qiskit, Cirq, CUDA Quantum, Qaptiva, Pennylane, IQM Pulse; TKET, OpenQASM (in preparation)
Cooling	Cryogenic (dilution refrigeration).
Supplier	IQM.

Table 4: Summary highlights of the VLQ quantum computer at IT4Innovations.

4.3. Access to specific quantum hardware and simulators

For users with high-level research needs, DeiC offers access to specific quantum hardware as well as the corresponding hardware-specific simulators. To participate, researchers must apply during through Call for expression of interest in access to quantum computing resources.

5. UCloud Allocation for users affiliated with Danish universities

To obtain CPU and GPU resource for smooth quantum simulation and pre-quantum hardware design for various innovation and research projects, users **affiliated with Danish Universities** can apply corresponding resource via **UCloud**, which already integrates CUDA-Q (NVIDIA's GPU-based quantum simulator). For example, through the insights gathered from quantum simulators one can build better models or extract new understanding of the modelled materials as shown in Figure 21 [26].

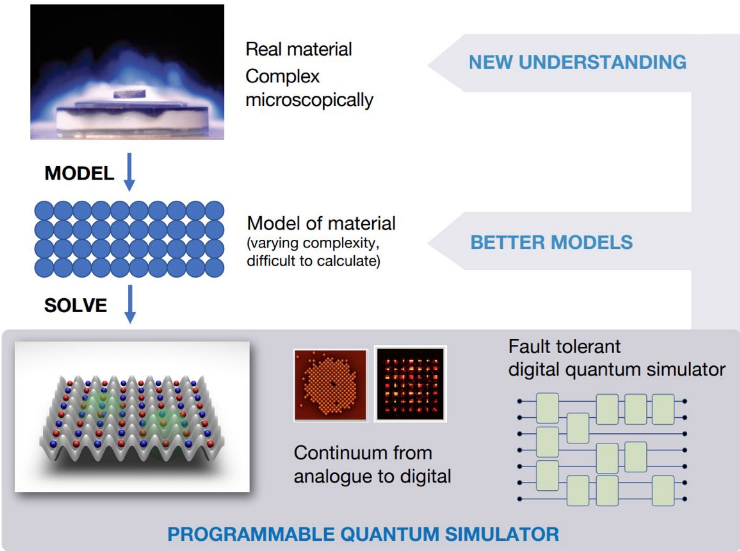


Figure 21: Practical quantum advantage in quantum simulation to study physical properties of real and promising materials.

5.1. Why Use UCloud for Quantum Simulation?

Using the CUDA-Q application interface on UCloud expand the coverage of researchers' needs in Denmark enormously, because it

1. **Expands backend compatibility** (see picture below)

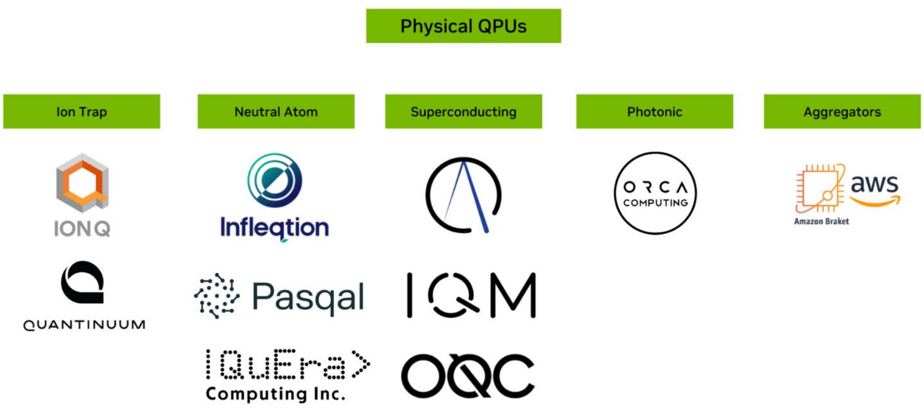


Figure 22: CUDA-Q supports submission to a set of hardware providers. Source: NVIDIA

- Supports non-gate based quantum systems, including quantum annealing and boson sampling, which are completely uncovered by Microsoft Azure.
- Contains additional gate-based quantum systems with different modalities.

2. **Enables efficient large-scale simulation**:
   - GPU-based simulation significantly outperforms traditional CPU-based simulators. For quantum circuits exceeding 20 qubits or involving moderate to deep circuit depths, the later becomes impractically slow or entirely infeasible.
   - While many quantum providers and aggregate platforms (e.g., Azure Quantum, AWS Braket) offer free simulation, these typically utilize CPU backend, which constrain the complexity of algorithms that researchers can realistically test.
3. **Fits more research need**:
   - Many theoretical studies can be done entirely in simulation.
   - Large scale quantum-classical hybrid algorithm design and tests need efficient simulation as pretesting before operating on quantum computers.
   - Conduct meaningful benchmarking and explore error analysis.

5.2. Judging computational resource need for quantum simulation

The following graph provides loose guidelines to choose suitable hardware (classical or quantum) for quantum simulation (This section is adapted from the *quantum simulation guide published by Google* [27]).

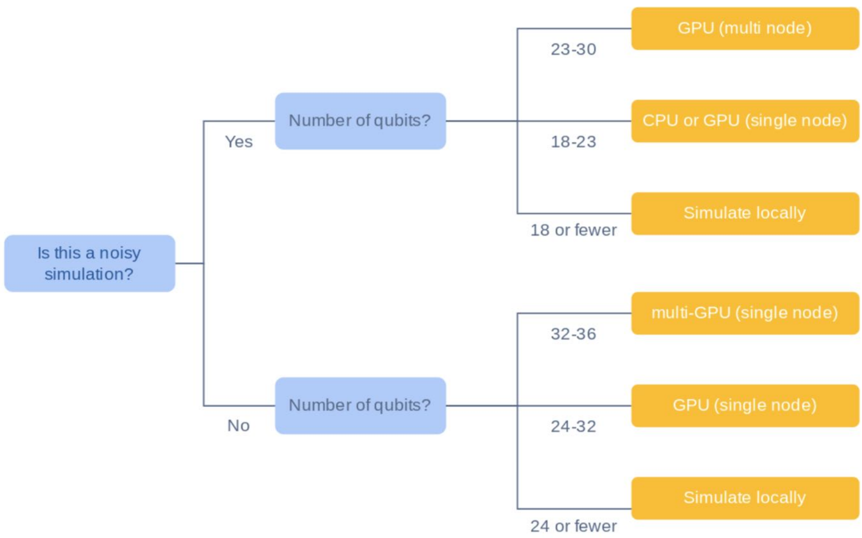


Figure 23: Resource selection for quantum simulation

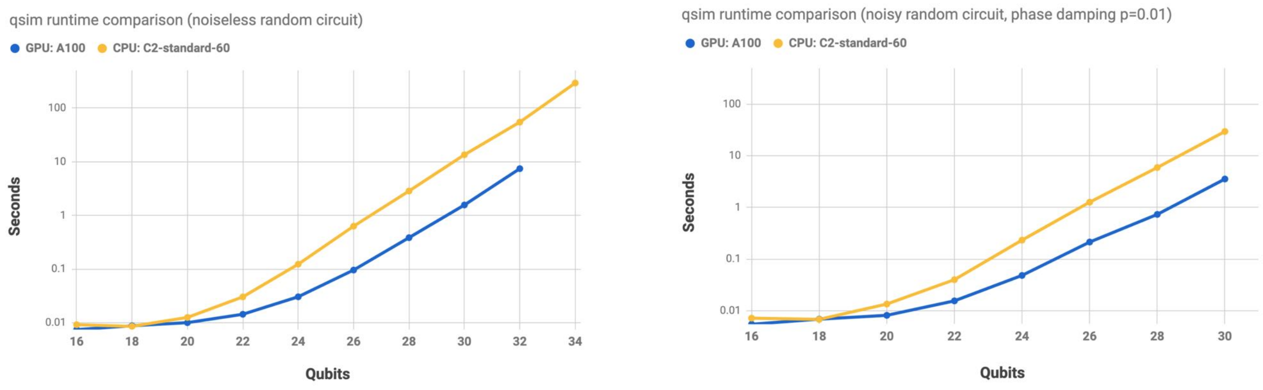
The increasing of the size and complexity of quantum simulation rapidly demands a large increase in computational power, which depends on the following parameters:

- Noise; noisy simulations require more compute power than noiseless simulations.

- Number of qubits.
- Circuit depth; the number of time steps required to perform the circuit.

Deciding among CPUs, single GPUs, or multiple GPUs

GPU hardware starts to outperform CPU hardware significantly for circuits with more than 20 qubits. The maximum number of qubits that one can simulate with a GPU is limited by the memory of the GPU. For example, for a noiseless simulation on an NVIDIA A100 GPU with 40GB of memory, the maximum allowed number of qubits is 32.



(a) Noiseless Simulation

(b) Noisy Simulation

Figure 24: Comparison of noiseless and noisy quantum simulations in log scale.

The above two charts demonstrate the runtime for a random circuit for the noiseless and noisy simulation, respectively. The computation is run on Google Compute Engine, using an NVIDIA A100 GPU, and a compute-optimized CPU (c2-standard-4).

Multi-GPU simulations are supported by NVIDIA's cuQuantum Appliance, which is implemented by UCloud as well.

5.3. How to apply computational resource on UCloud

The instruction below applies primarily to SDU-affiliated users regarding general application regimes that demand GPU resources (which naturally cover quantum computing simulation) on UCloud. DeiC's quantum department is currently coordinating with HPC front offices at other Danish universities to display the process for general cases.

Criteria for regular and large scale projects

According to SDU policy, projects requiring more than 50,000 CPU hours, 1,000 GPU hours, or 50 TB of storage (i.e., a large project) will undergo evaluation twice a year following the deadlines of calls for national HPC resources. A prerequisite is that the same application must also be submitted to the national DeiC call. Regular projects below the above threshold are typically reviewed within 14 days.

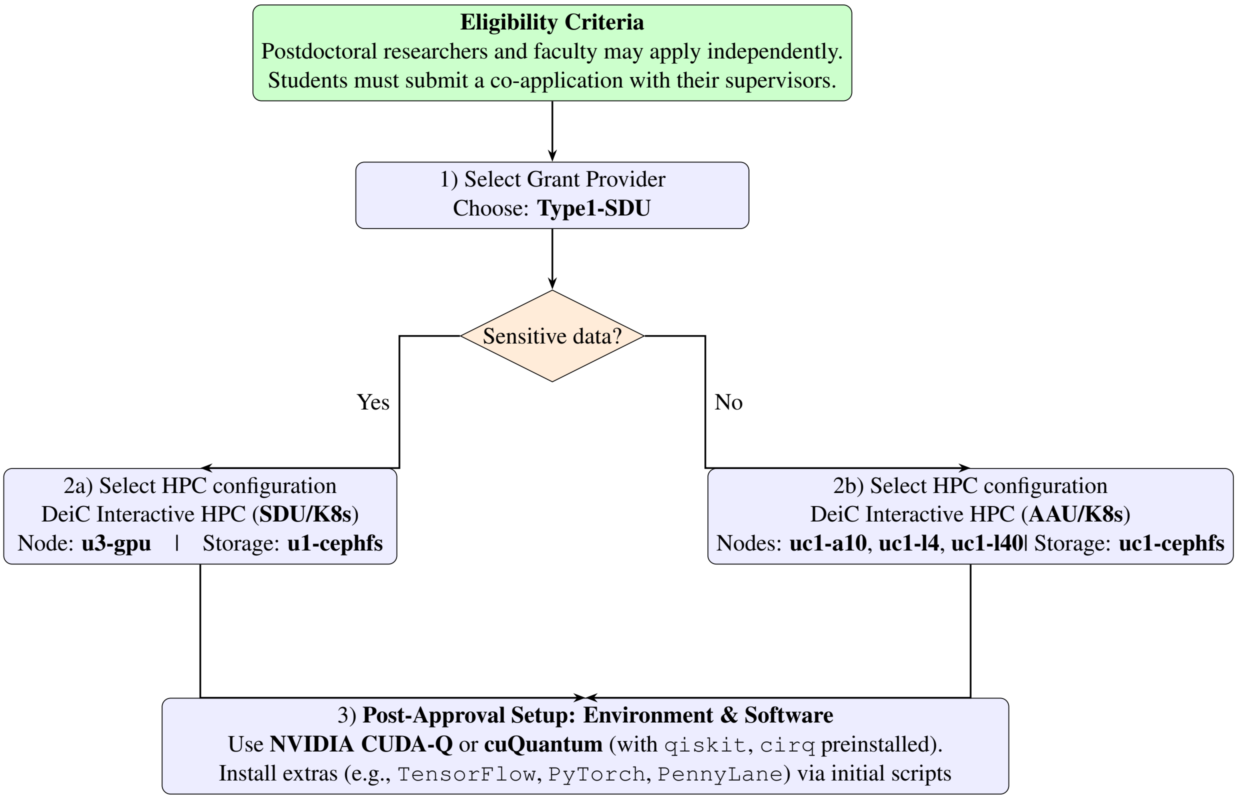


Figure 25: Flowchart: Eligibility, applying for GPU hours, and post-approval environment setup.

Post-Approval Setup: Environment & Software

Software Stack Preparation Researchers are encouraged to use the NVIDIA CUDA-Q and cuQuantum platforms, which come with preinstalled standard libraries such as numpy, pandas, OpenMPI, CUDA, qiskit, and cirq,etc. Additional packages like TensorFlow, PyTorch, or PennyLane can be installed via initialization scripts (DeiC is preparing a standardized initializing script that covers the majority of needs now).

Access to Real Quantum Computers (Subject to Legal Review by DeiC)

- CUDA-Q allows access to quantum hardware via credential-based integrations.
- For example, to access Quantinuum's device, users must provide a verified account and password registered with Quantinuum.

6. Metrics and Benchmarks of Quantum Computers

Quantum computers are increasingly accessible to both academic and industrial users, it is therefore important to establish transparent methods with standardized benchmarks to evaluate the quantum resources being used [29, 30]. This section is mainly adapted from the published work [31], which provides guidance for deriving holistic comparisons of quantum computers across dimensions such as hardware properties, computational quality and execution speed.

We select three major categories of evaluation metrics, aligned to the "Three Key Metrics" proposed by IBM in Section 6.4 for assessing quantum devices. A broader and more comprehensive set of metrics is schematically illustrated in Fig. 26.

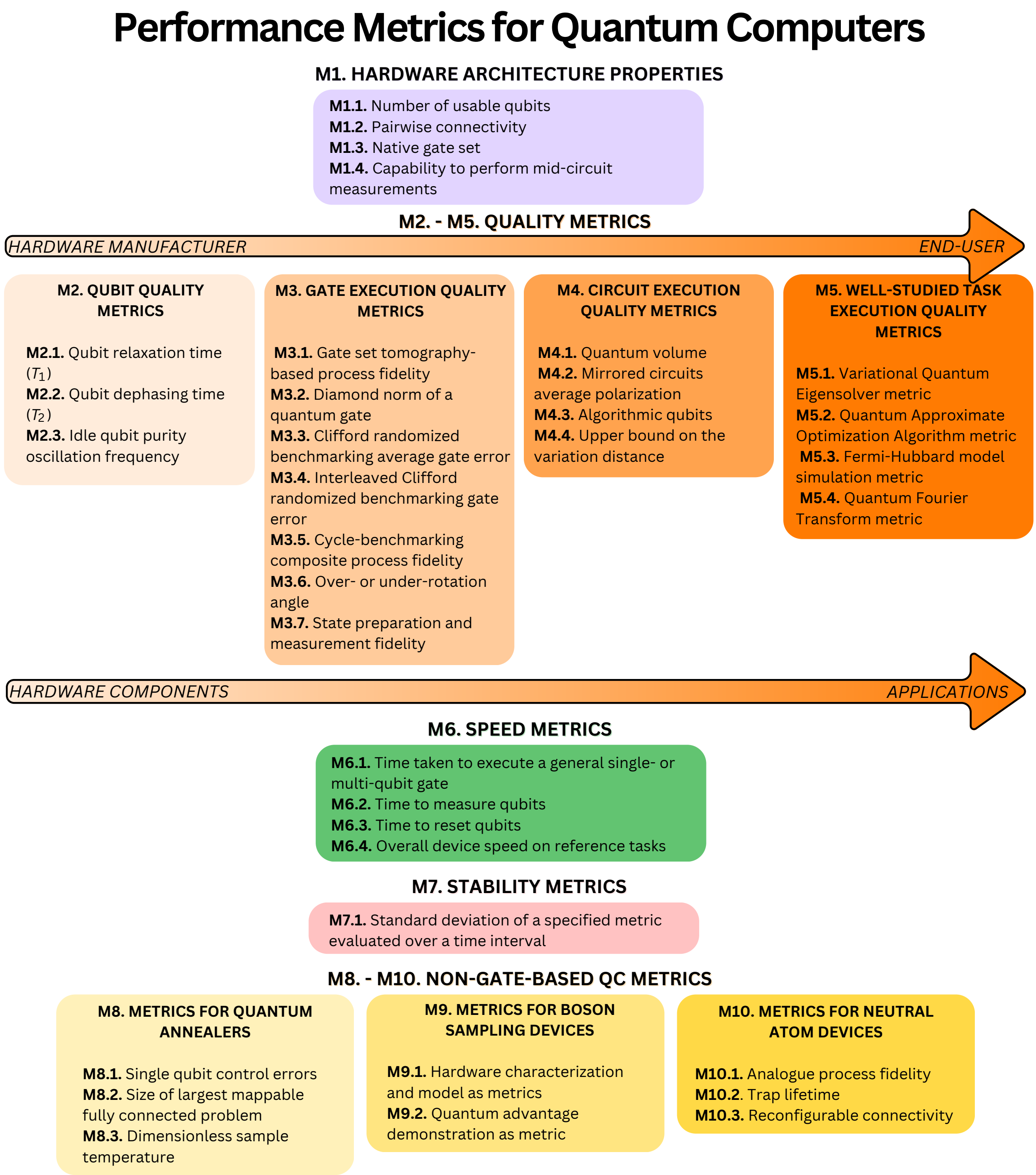


Figure 26: Schematic collection of metrics. [31]

6.1. Hardware metrics

The hardware metrics determine the general capabilities of the device, including:

1. **Number of usable qubits** In particular, we use the term physical qubits for the actual qubits present in the hardware, while the virtual error-resilient ones are named logical qubits. A collection of road map toward scalable universal devices from leading quantum computing company is displayed in Fig 27.

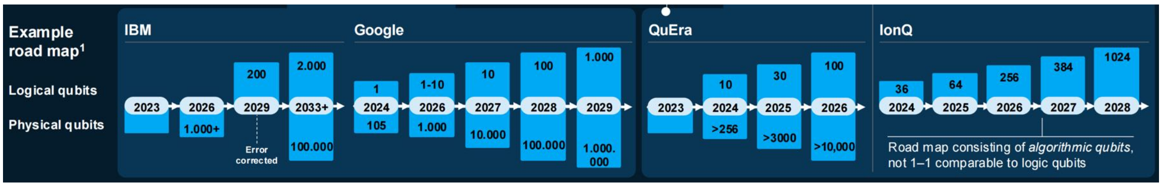


Figure 27: Road maps toward scalable universal devices, source: McKinsey Digital

2. **Pairwise Connectivity** Traditional qubits arrangement typically employs a structured 2D lattice, which facilitates easy manufacturing and scalability. However, such configurations are inherently constrained in terms of entangling compared to more advanced designs that support *all-to-all* connectivity, where any pair of qubits can interact directly. Table 5 and Figure 28 illustrate the comparison between QPUs based on conventional lattice architectures and those adapt advanced topologies.

Feature	Traditional lattice QPU	Advanced QPU Topology
Qubit Connectivity	Limited (nearest neighbors)	All-to-all
Need for SWAP Gates	High (adds noise)	Low (direct interactions)

Table 5: Comparison of connectivity between traditional 2D structured qubit arrangements and advanced topologies.

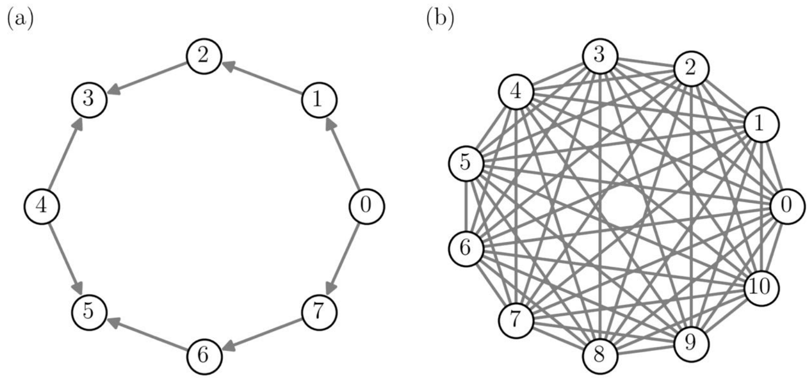


Figure 28: The connectivity of (a) the OQC superconducting circuit quantum computer "Lucy" and (b) the IonQ trapped-ion quantum computer "Harmony". [31]

3. **Native Gate Set** A universal quantum computer must be able to perform arbitrary unitary operations. Any multi-qubit unitary can be decomposed into a circuit of general single-qubit and two-qubit gates [32,33]. Therefore, it is essential to clarify the set of single and two-qubit gates, such that the hardware platform can implement natively [34,35].

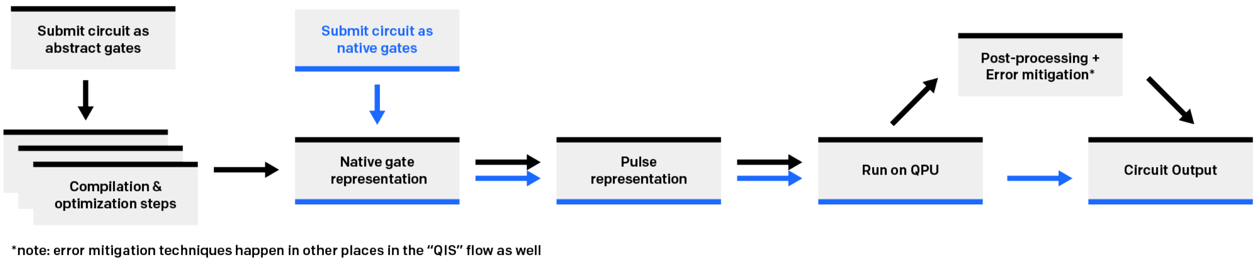


Figure 29: Demonstration of native gate flow. Source: IonQ.

4. **Capability to perform mid-circuit measurements** It refers to measure and then reset qubits multiple times within a single quantum circuit execution. Many quantum algorithms, including quantum error correction, demand this action heavily to meet the resource requirements of the algorithms [36–42].

6.2. Quality metrics

The quality metrics quantify the error rates of quantum computing hardware, which is achieved by calculating various differences of the outputs of the physical hardware and those expected for an ideal noise-free quantum computer. Quantifying the quality is one of the most challenging aspects [43], since there are enormous sources of errors, and these sources can be highly distinct for different hardware platforms.

6.2.1. Qubit and quantum gate quality metrics

Qubits are not isolated from their environment, and their interaction with the environment can significantly perturb their quantum state, leading to the loss of information [32, 44–50].

Qubit relaxation time (T1) and Qubit dephasing time (T2)

- T1 — the time taken for the excited $|1\rangle$ state decays to the ground state $|0\rangle$.
- T2 — Dephasing affects the superposition phase.

These metrics are central in quantifying the duration over which a qubit preserves its state and stays coherent. Therefore, the number of gates that can be successfully applied is determined by the ratio between these times and the duration of a quantum gate. Different hardware platforms can have gate durations that differ by orders of magnitude [51–55]. For example, ion-trap based devices typically have orders of magnitude longer coherence times than devices such as superconducting qubits, but also typically have much longer gate durations [54].

Gate execution quality Often, there is a trade-off between the gate control parameters: longer gate durations can allow reducing the effects of leakage but come at the cost of increased qubit decoherence during gate execution [56, 57]. Therefore, the gate fidelity index is designed to quantify the difference between the ideal noise-free gate and the gate executed on hardware.

	Ions	Superconductors	Neutral atoms
Qubit T1 coherence time	10-100 s	15-30 µs	7.6 s
Qubit T2 coherence time	1000 ms	30 µs	1500 ms
Two-qubit gate duration	0.6 ms	12 ns	0.25 ms
Number of operations	1667	2500	6000

Figure 30: Compare qubit and gate quality for the major quantum computing platforms: Ions, superconducting systems, and neutral atoms. Source: Guest post by Alex Lukin, Tommaso Macri, QuEra Computing Inc.

6.2.2. Circuit execution quality metrics

The effect of individual gate errors on the full circuit execution result is difficult to estimate, since each quantum circuit amplifies individual gate errors in a different way. Instead, to evaluate the quality of circuit execution one can use metrics that characterize the quality of execution of entire circuits directly [58–61].

Quantum volume is dominantly used to evaluate the performance of quantum computing. It is calculated by running randomized square circuits, where the number of circuit layers are equal to the number of qubits, for increasing numbers of qubits. Thus the quantum volume is evaluated by finding the largest square circuit that can be run on the device and that also passes an acceptance criterion based on classical simulation of the same quantum circuits.

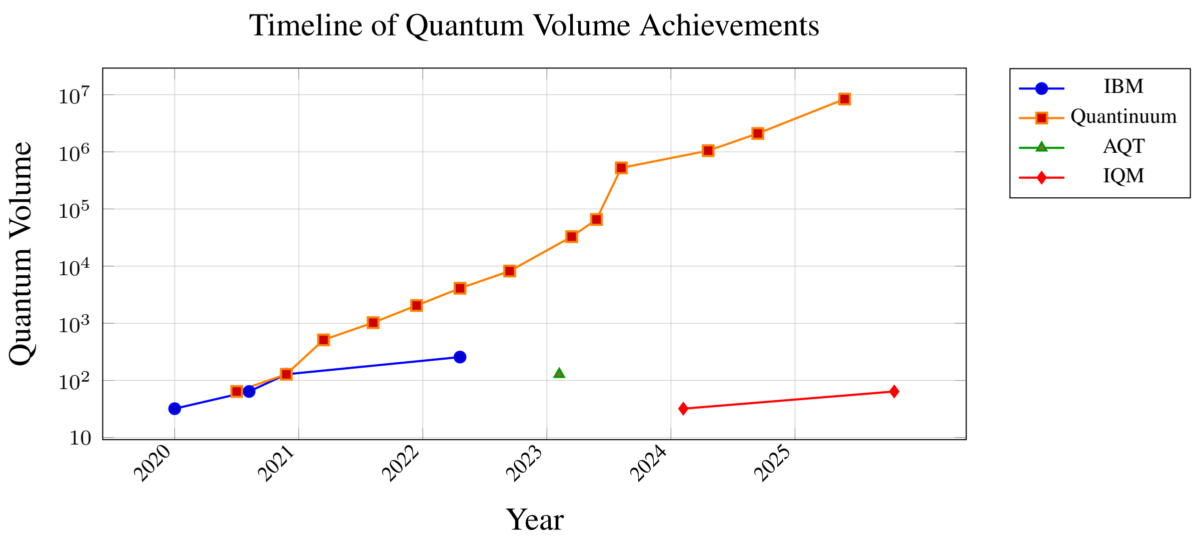


Figure 31: Development of Quantum Volume (QV) across major providers. Source: *Wikipedia: Quantum Volume Achievement History*.

6.3. Speed metrics

Variations in the device speed over orders of magnitude on the hardware platform [54] can make the difference between an algorithm finishing in a day or within many years of runtime. In 2021, IBM defined a speed metric: circuit layer operations per second (CLOPS) as a measure of how quickly processors can run quantum volume circuits in series.

Provider / System	CLOPS (layers/sec)	Source Date
IBM (Heron R3)	250k	2025
IQM	2600	2024

Table 6: Circuit Layer Operations Per Second (CLOPS) reported by major quantum computing providers. Today, IBM defines CLOPs by many layers of a 100x100 circuit (hardware-aware circuit) a QPU (quantum processing unit) can execute per unit of time.

6.4. Summary - Three key metrics for measuring quantum devices

For a simple and quick evaluation, end users may adopt the multi-dimensional benchmarking framework proposed by IBM, which considers selected key metrics: *scale*, *quality*, and *speed* to compare quantum systems across different architectures and performance levels.

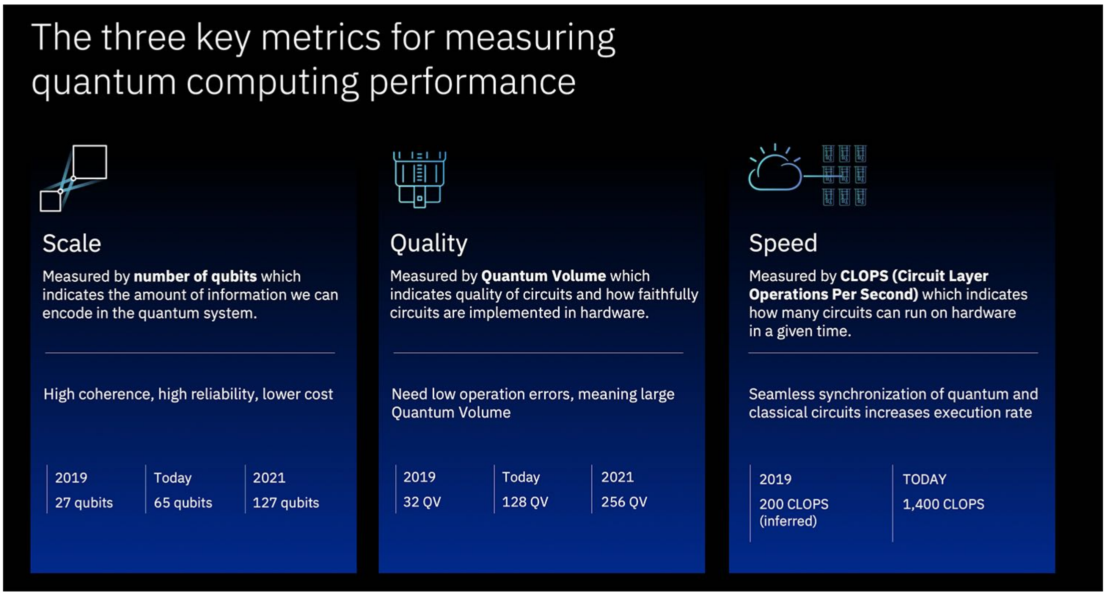


Figure 32: Three Key metrics for measuring quantum devices. Source: IBM.

However, as discussed in Ref. [31], a truly comprehensive benchmark assessment should incorporate a broader set of evaluation criteria. Rather than focusing solely on the three dimensions of scale, quality and speed proposed by IBM, one must also evaluate architectural properties (such as pairwise connectivity and native gate set), gate-level quality (e.g., qubit relaxation and dephasing time, gate set tomography-based process fidelity, etc) and stability over time (standard deviation of a specified metric evaluated over a time interva).

7. Types of Quantum Computers and How They Work

The most widely adopted model of quantum computation is denoted as gate-based quantum computing, where a discrete set of unitary operations, denoted as gates, is applied to perform the computation. In contrast, non-gate-based quantum computing approaches are typically tailored to exploit the specific strengths of each hardware platform. Examples include quantum annealing [62–64], boson sampling [65–68], and analogue quantum simulation [69–72].

7.1. How to create qubits

Generally, qubits are created by manipulating systems that exhibit quantum mechanical behavior, including superconducting circuits [73–80], trapped ions [81–88], neutral atoms [89–95], as well as photonic [96–102], and semiconductor devices [103–109], each better suited for different tasks.

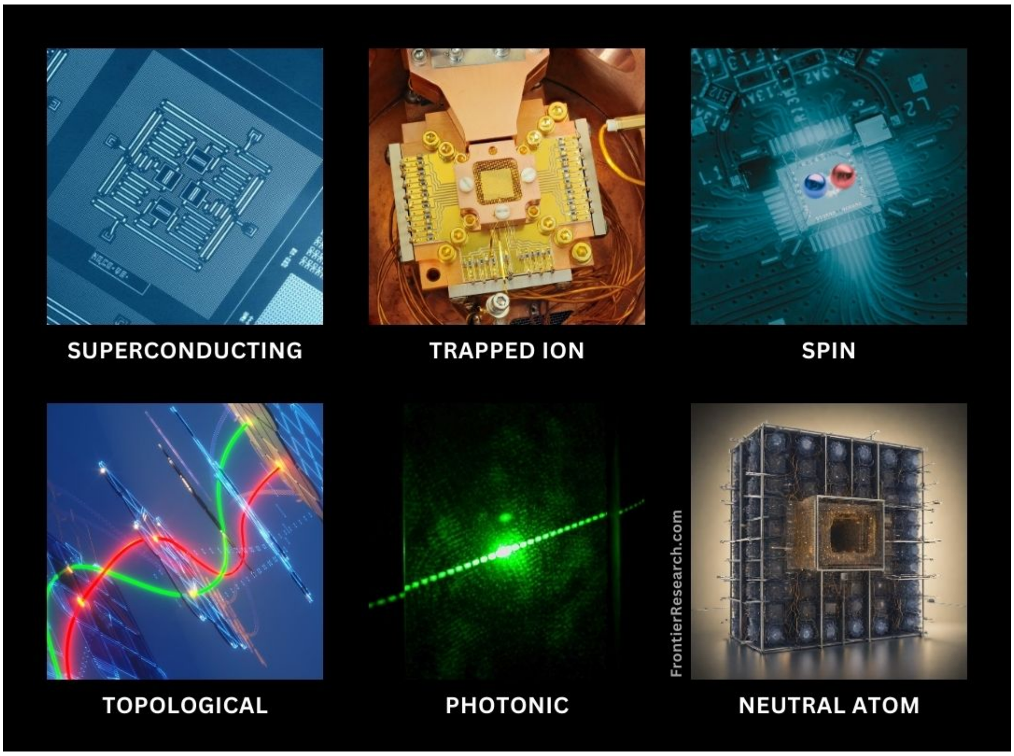


Figure 33: Most common types of qubit technologies. Source: exoswan, 2025.

7.2. Introduction to Common Types of Quantum Computers

Today, numerous companies and organizations are striving to establish their architecture as the leading approach and to be the first to demonstrate practical utility or quantum advantage on a large-scale quantum device.

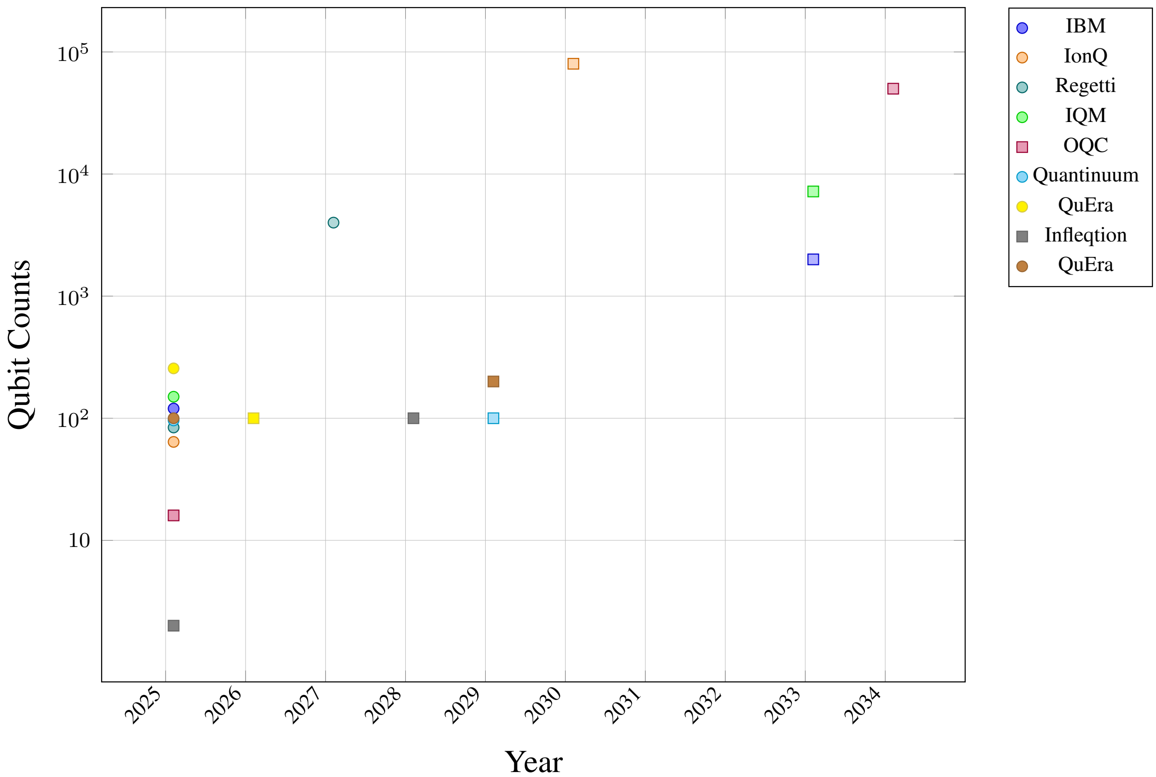
This chart below summarizes public roadmaps and indicative targets across major quantum hardware providers. Each marker shows an announced or projected milestone, including qubit counts (physical or logical as stated). Note that timelines and capacities are forward-looking and may change as vendors update specifications later; treat them as indicative rather than guaranteed commitments.

Readers who wish to explore more detailed information, including error-corrected targets, gate counts, fidelities, and related specifications, are encouraged to deep dive the corresponding development roadmap of each provider in this section.

Logical qubit threshold

There is currently no universal definition of what constitutes a *logical qubit*. Different hardware roadmaps adopt their own threshold criteria depending on the targeted logical error rate (typically below 10^{-5}). Consequently, the reported logical qubit counts in the diagram should be interpreted as indicative milestones rather than directly comparable values across platforms.

Figure 34: Development of qubit counts across major providers: physical qubits (2025) vs. logical-qubit roadmap endpoints. Circles = physical; squares = logical.



From now on, we will provide an overview of the each specific type of qubit construction, explaining their operating principles and other key features for both general and professional audience.

7.2.1. Superconducting

The 2025 Nobel Prize in Physics was awarded to John Clarke, Michel H. Devoret, and John M. Martinis for their pioneering experiments in superconducting circuit that demonstrates both quantum tunnelling and quantised energy levels in a macroscopic system — large enough to be held in the hand. This groundbreaking achievement laid the foundation for many quantum hardware technologies such as quantum computers and quantum sensors.

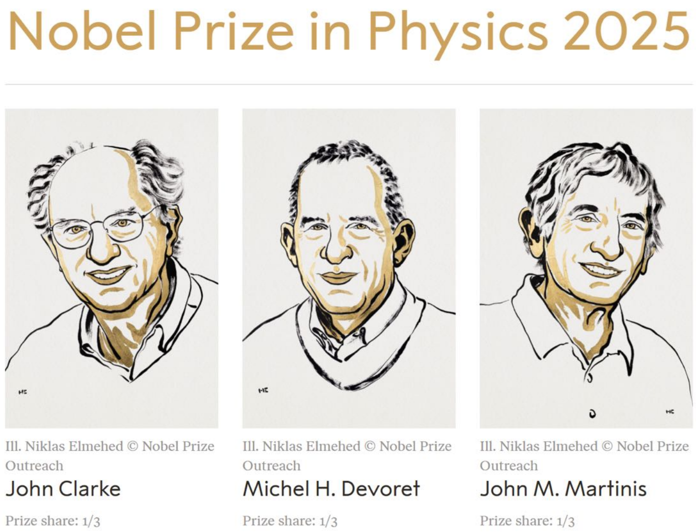


Figure 35: The Nobel Prize in Physics 2025 was awarded jointly to John Clarke, Michel H. Devoret and John M. Martinis "for the discovery of macroscopic quantum mechanical tunnelling and energy quantisation in an electric circuit", Source: Nobelprize.org.

Background: Cooper pairs behave as a single quantum mechanical unit

In ordinary material, electrons (carrier particles of electric charge) move as individual particles and experience electrical resistance. When a material becomes superconducting, however, the electrons form bound pairs known as *Cooper pairs* (Nobel Prize in Physics 1972).

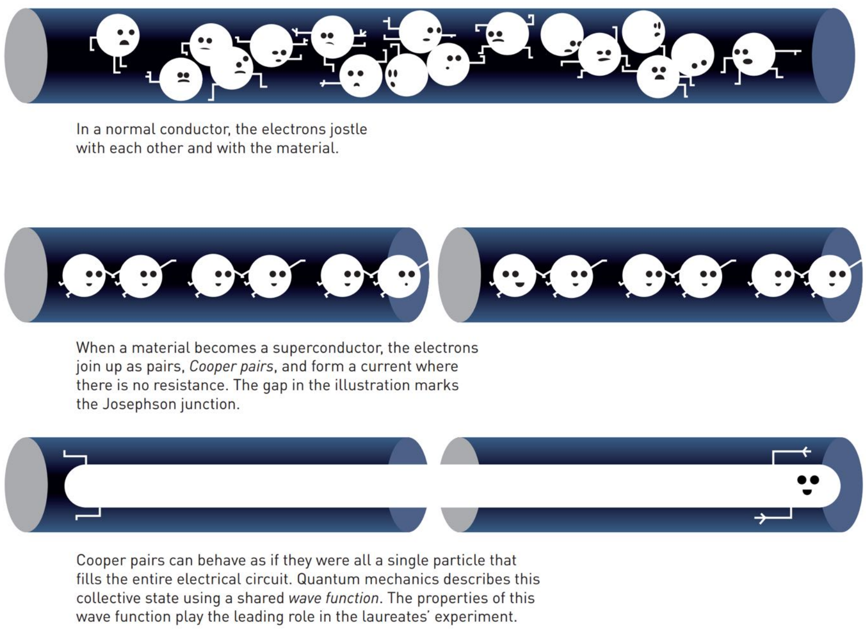


Figure 36: Cooper pairs behave as a single quantum mechanical unit. Source: Johan Jarnestad/The Royal Swedish Academy of Sciences.

When two superconductors are connected by a thin insulating barrier, they form a Josephson junction — illustrated as a narrow gap in the diagram below. This component, named after Brian Josephson, realizes the Josephson effect, which describes the properties of a supercurrent through a tunnel barrier. (Nobel Prize in Physics 1973). Even though the barrier is non-conducting for electrons, Cooper pairs can quantum-tunnel through it when a weak current is applied to the Josephson junction. This setup allows the system to tunnel from its original state to quantized energy levels with a certain probability. Those observations clearly demonstrate that the Cooper pairs act as one large quantum particle.

Formation of a superconducting qubit

A superconducting circuit exhibiting quantised states can therefore serve as a fundamental unit of quantum information — a *qubit*. The lowest energy state and the first excited state represent the qubit states $|0\rangle$ and $|1\rangle$, respectively. By applying microwave photons to these qubits, their state can be precisely manipulated, enabling operations such as initialization, control, and readout of individual qubits.

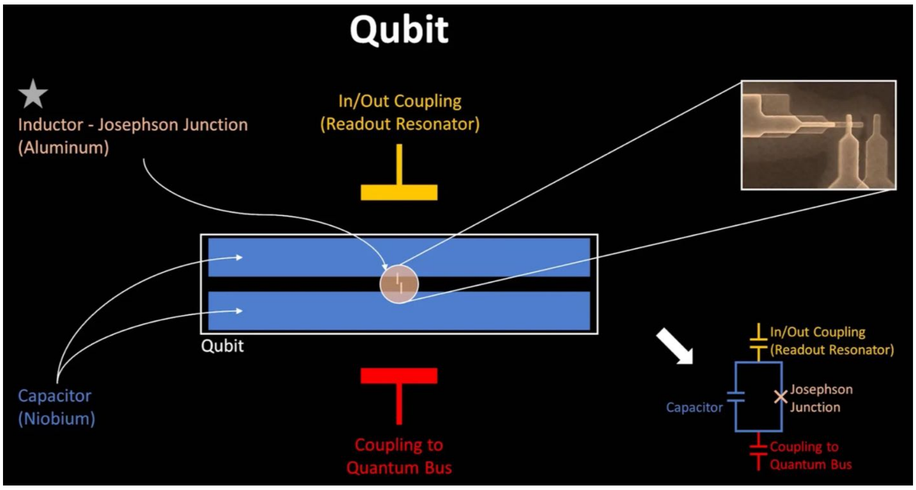


Figure 37: Formation of a qubit within a superconducting circuit. Source: IBM.

Leading quantum hardware providers and their roadmaps

IBM

IBM Quantum offers one of the largest cloud-accessible superconducting qubit platforms, with systems available under per-minute plan tiers ($48–$96/min). In 2029, they aim to deliver a system that accurately runs 100 million gates on 200 logical qubits — unlocking the first viable path to realizing the full power of quantum computing.

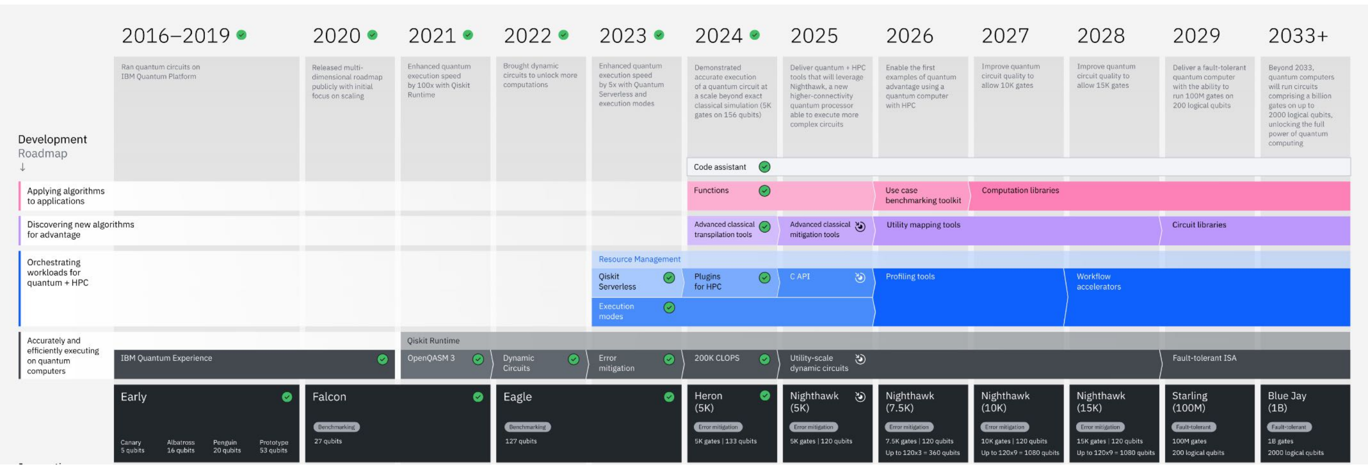


Figure 38: 2025 Development Roadmap, IBM Quantum.

IBM and *Qiskit*

In addition, as a full-stack quantum computing provider, IBM not only develops superconducting quantum hardware but also maintains the world's largest quantum programming and user ecosystem through its open-source software framework, *Qiskit*.

This integrated platform supports algorithm development, simulation, and deployment on real quantum processors, thereby fostering a global community of researchers, educators, and industry users. Through Qiskit and the IBM Quantum services, a broad range of applications, spanning chemistry, finance, optimization, and machine learning, have been demonstrated, as summarized in Table 7.

Sector	End Users	Description
Pharmacy	Moderna	Apply variational quantum algorithms (VQAs) and Conditional Value at Risk (CVaR) to the mRNA secondary structure prediction. (Source)
Aerospace	Boeing	Quantum optimization on engineering strong, lightweight materials (ply composite). (Source)
Chemistry	Mitsubishi Chemical	Create and study accurate molecular simulations of complex electrochemical reaction for lithium superoxide rearrangement in lithium-oxygen batteries. (Source)
Electrical Car	Mercedes-Benz	Next generation of batteries technologies – quite possibly the lithium-sulfur (Li-S) battery towards a carbon-neutral new passenger car fleet. (Source)
Semiconductor	JSR	Aid computer chemical simulations of photo-acid generators (PAGs) to develop new photoresists more quickly and at lower cost. (Source)
Energy	E.ON	Energy price decision under weather-related risk modeling. (Source)
Routing	ExxonMobil	Model maritime inventory routing, analyze the strengths and trade-offs of different strategies to efficiently transport Liquefied Natural Gas (LNG). (Source)
Denmark	DTU, KU & SDU	Develop quantum solutions to study molecular properties—such as metalloenzymes and light-harvesting chromophores in proteins. (Granted DKK 40 million by Novo Nordisk Foundation) (Source)

Table 7: IBM Quantum End Users by Industry

Rigetti

Rigetti processors leverage its distinctive Fab-1 capabilities to achieve both performance and scalability. The Rigetti-developed quantum language 'Quil' combines gate-level and pulse-level control of quantum circuits while integrating classical instructions and shared memory. User can access through a pay-as-you-go model at $0.02 per 10 millisecond.

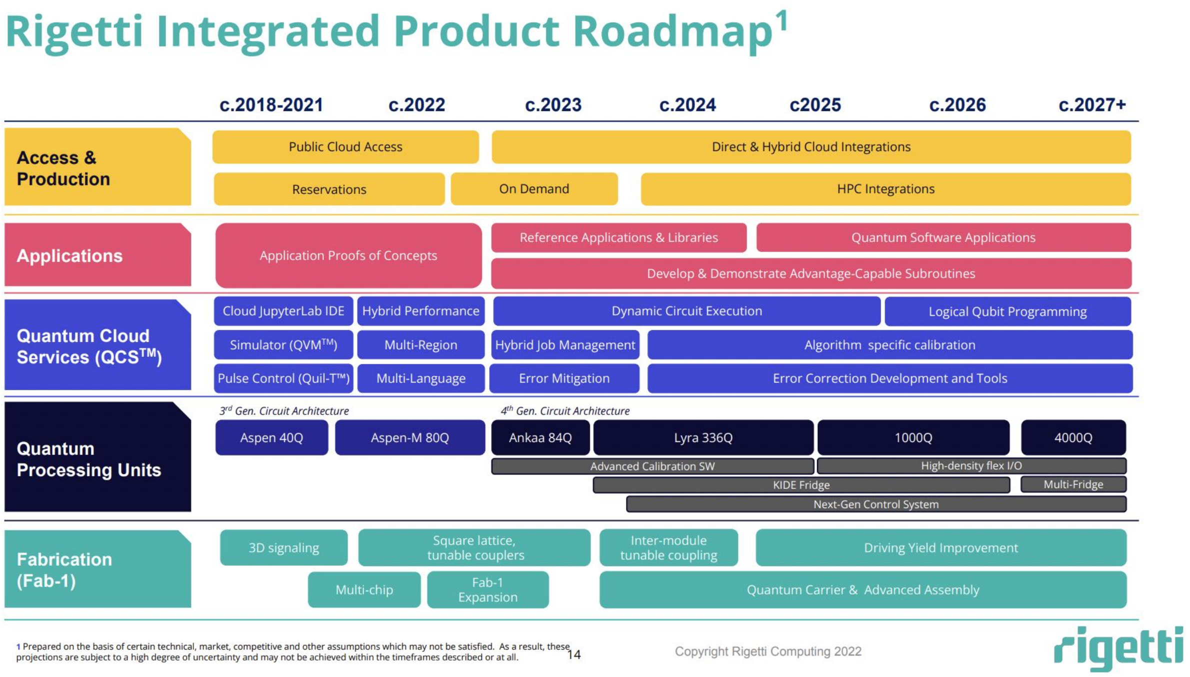


Figure 39: 2022 Development Roadmap, Rigetti.

Sector	End Users	Description
Benchmark	QuAIL, NASA & RIACS	Develop benchmarking of quantum algorithms (e.g., QAOA), applied to the MaxCut problem, to quantify the quantum advantage over classical computation (Source).
Research	Standford University	Develop incentive-based programming of hybrid quantum-classical computing systems using reinforcement learning, and apply this to solve combinatorial optimization problems (Source) .
Music	ICCMR	Development of a singing voice synthesiser and a musical sequencer based on quantum walk for creating, performing, listening to and distributing music (Source) .

Table 8: Rigetti End Users by Industry (continued)

IQM

IQM focuses quantum ecosystem with scalable hardware, advanced software integrations for HPCs, and application development. Users can access through a pay-as-you-go model starting at $0.30 per second, or opt for a premier plan with additional pulse-level control (available upon request for customized pricing). Notably, IQM has achieved the milestone of delivering more quantum computers than any other company over the past twelve months .

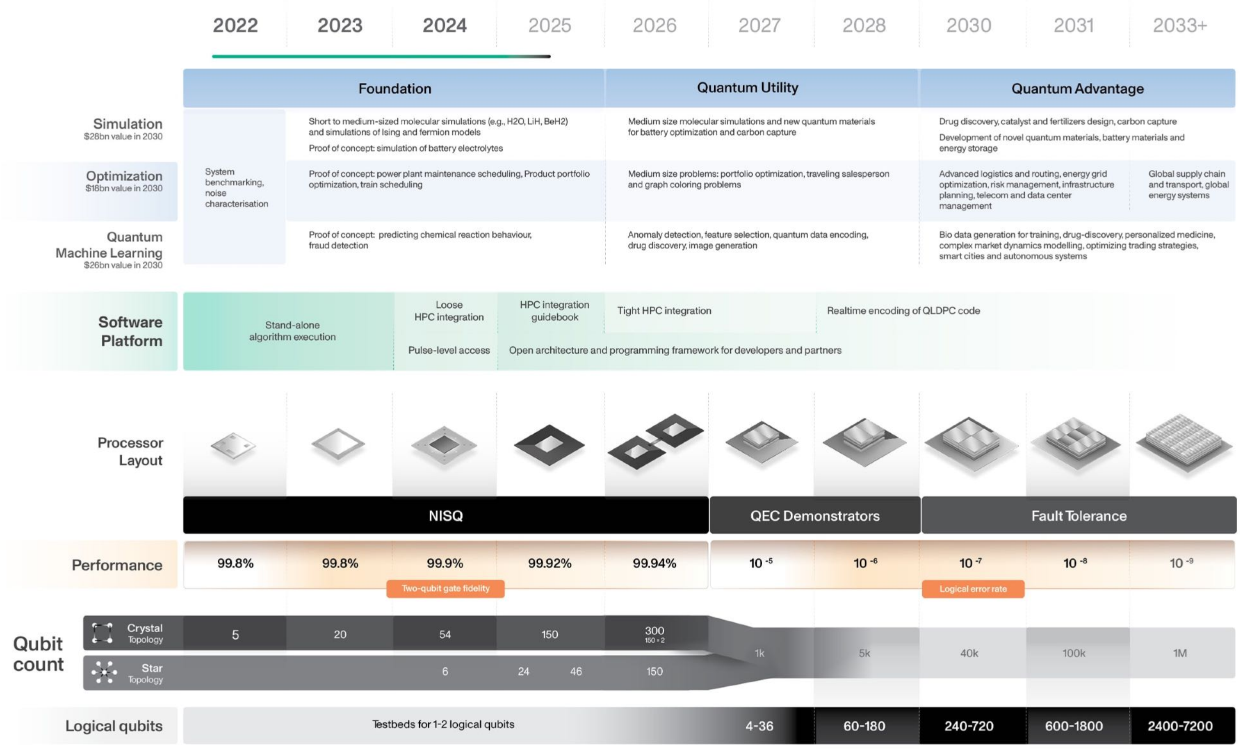


Figure 40: 2025 Development Roadmap, IQM.

Sector	**End Users**	**Description**
Trading	DATEV 🇩🇪	Quantum solution to handle the complexity and scale of portfolio optimization on the selection of assets, in order to maximizing returns and minimizing risk (Source) .
R& D	VTT 🇫🇮	A joint project with VTT (a limited liability company owned by the Finnish state) to develop quantum infrastructure. The access to the device is opens for use by companies and researchers through VTT quantum computing service (Source) .
Chemistry	Volkswagen 🇩🇪	Apply trial wavefunctions on quantum devices to reduce the bias of auxiliary-field quantum Monte Carlo (QC-AFQMC). This hybrid approach to the simulation of strongly correlated many body systems in battery chemistry reduces the required number of qubits while preserves accuracy (Source) .
HPC Integration	IQM–LUMI-Q 🇪🇺	LUMI-Q is a consortium consisting of European quantum stakeholders formed as an initiative to establish and strengthen collaborative ties within the quantum technology sector. The quantum computer is integrated with HPC (LUMI), see LUMI-Q subsection for details (Source) .

Table 9: IQM Quantum End Users by Industry

Anyon Technologies

Anyon Technologies is a Singapore- and California-based quantum computing company that has been at the forefront of developing highly scalable modular quantum computing systems based on cryogenics optimized for quantum computing. Augmented by their integration with the world's most powerful GPU technologies, they powers high-impact enterprise applications in finance, artificial intelligence, and chemical engineering today and tomorrow at scale.

Sector	End User	Description
Hardware	YQuantum 🇨🇭	Construct next generation quantum computer based on YQuantum's expertise in scalable cryogenic components for quantum signal routing and filtering (Source) .
AI	SDT Inc. 🇰🇷 & MIMOS 🇲🇾	Establish Quantum Intelligence Centre for quantum-enhanced applications by supplying hybrid quantum classical computing infrastructure, integrating with NVIDIA GPU-accelerated computing technologies (Source) .

Table 10: Anyon Technologies Use Cases by Sector

OQC (Oxford Quantum Circuits)

Oxford Quantum Circuits (OQC) is a UK-based quantum computing company developing superconducting qubit processors using its patented *Coaxmon* architecture. The Coaxmon design has a three-dimensional architecture that brings key componentry off-chip for vastly increased simplicity, flexibility, engineerability, and scalability. OQC currently provides cloud access to its systems through its own OQC Cloud as well as via Amazon Braket.

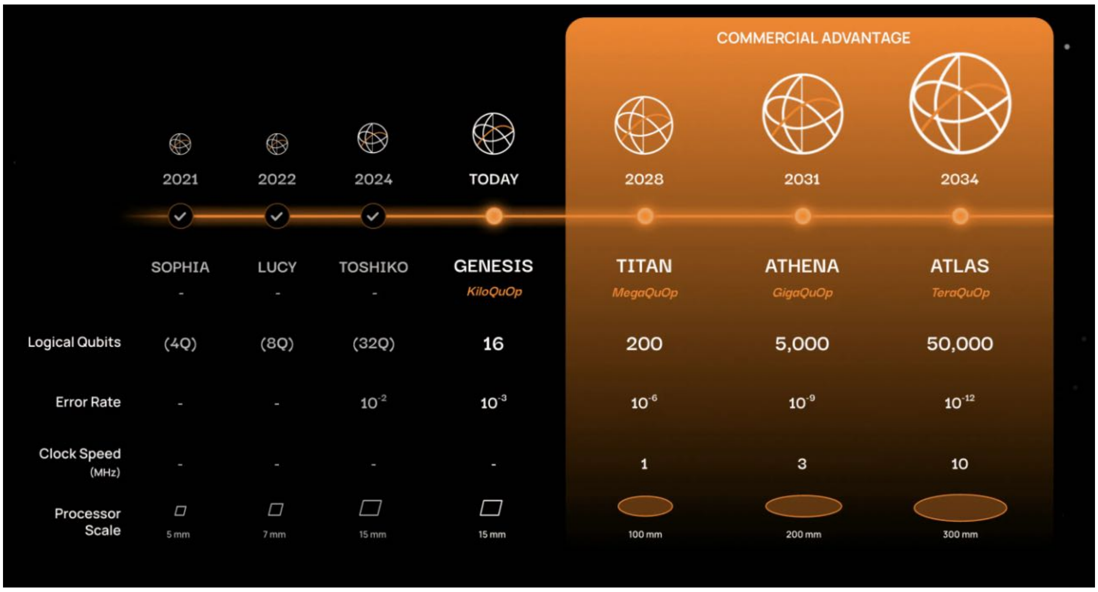


Figure 41: 2025 OQC Roadmap.

Sector	End User	Description
Transport	Department for Transport	Quantum optimization for both large-scale rail networks and detailed station routing, providing improved scheduling in transit time, robustness to delays, and reductions in operating costs and emissions (Source)
Chemistry	Kvantify	Integrate OQC's quantum hardware with Kvantify's proprietary quantum chemistry platform to model battery processes accurately, which is also aligned with UK's and Europe's green transition in achieving net zero by 2050 (Source)
Hardware	Riverlane	Build Quantum Error Corrected (QEC) testbed that incorporate hardware-efficient error correcting protocols. It is also integrated with HPC within a secure datacentre environment (Source)

Table 11: OQC End Users by Industry Sectors (continued)

7.2.2. Trapped Ions

In fact, the origin of ion-trap technology dates back to 1953, when Wolfgang Paul proposed the use of oscillating electric fields to confine ions with high precision. This pioneering concept, later realized as the *Paul trap*, made it possible to isolate and manipulate single ions for extended periods without contact. For this breakthrough, Hans G. Dehmelt and Wolfgang Paul were awarded the 1989 Physics Nobel Prize.

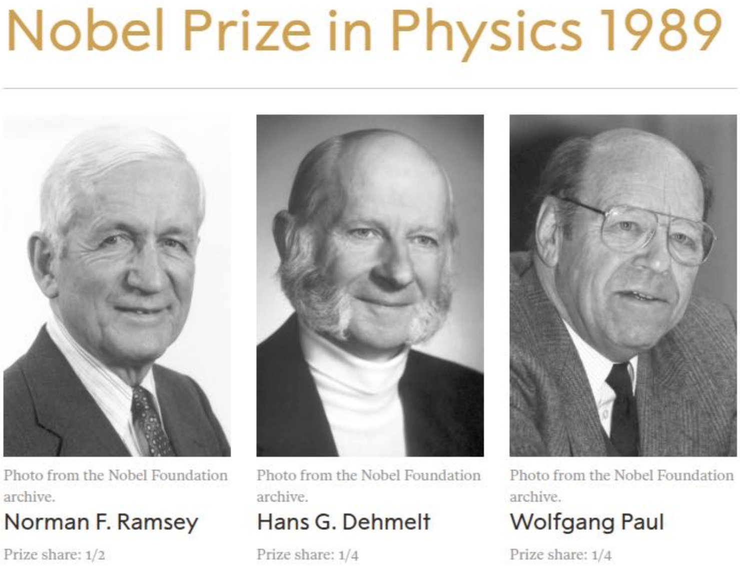


Figure 42: The Nobel Prize in Physics 1989 was divided, one half awarded to Norman F. Ramsey "for the invention of the separated oscillatory fields method and its use in the hydrogen maser and other atomic clocks" (key to quantum sensing), the other half jointly to Hans G. Dehmelt and Wolfgang Paul "for the development of the ion trap technique"

Properties of ion trapped systems today

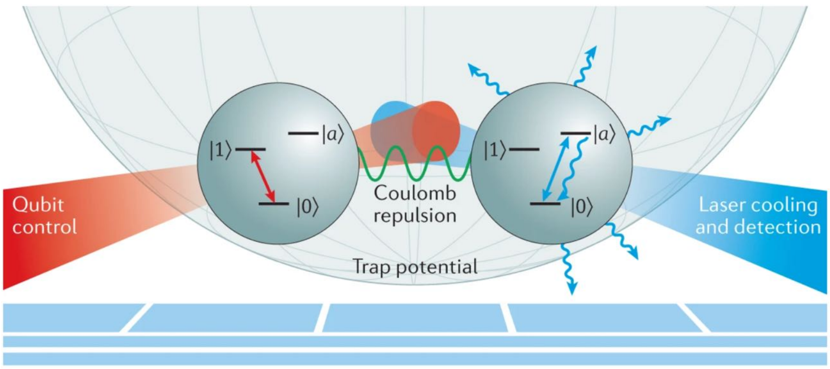


Figure 43: The ions are held in an electromagnetic trap. Lasers or microwaves are used to control the internal states of qubits, $|0\rangle$ and $|1\rangle$. The internal control and the Coulomb repulsion between ions combine to form conditional logic gates. Readout is performed by measuring laser-induced ion fluorescence using an auxiliary state $|a\rangle$ [111].

Today, trapped ion systems are characterized by a high degree of precise control over both their internal and external states [110]. The electronic structure of ions allows qubits isolated well from the environment, as well as a simple readout mechanism based on laser induced fluorescence.

Leading quantum hardware providers and their roadmaps

IonQ

IonQ was founded in 2015 by Chris Monroe and Jungsang Kim with seed funding from the University of Maryland and Duke University, taking the goal of transferring trapped ion quantum computing out of the lab and into the market.
In 2025, IonQ has announced its intention to acquire Oxford Ionics, accelerating the ability to deliver the world's most powerful fault-tolerant quantum computers with 2 million physical qubits and 80,000 logical qubits by 2030.

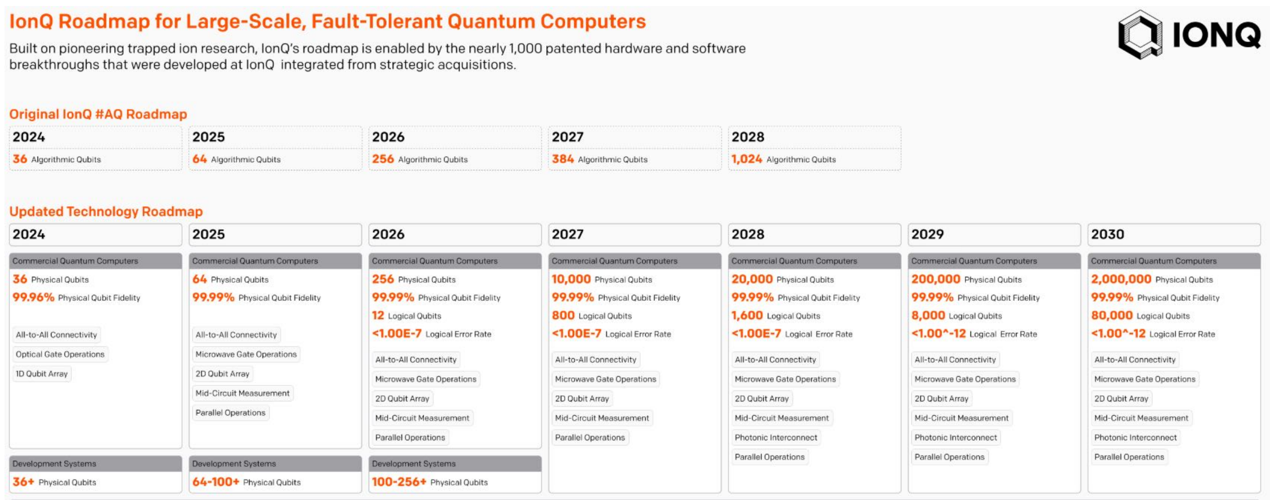


Figure 44: 2025 Development Roadmap, IonQ.

Sector	End Users	Description
Chemistry	Hyundai Motor Company 🇰🇷	Extend correlated sampling from classical auxiliary-field quantum Monte Carlo to the quantum-classical (QC-AFQMC) framework, enabling accurate nuclear force computations crucial for geometry optimization and reaction dynamics (Source) .
Energy	Oak Ridge National Laboratory 🇺🇸	Apply variational quantum algorithm (VQA) for the Unit Commitment (UC) problem in power systems which aims at minimizing the total cost while optimally allocating generating units to meet the hourly demand of the power loads (Source) .
Space	U.S. Department of Energy 🇺🇸	Design and execute an orbital demonstration of quantum-secure communications on satellite platform. Additional quantum applications in space also includes alternate position, navigation, and timing, time synchronization, quantum networking, and sensing (Source) .

Table 12: IonQ End Users by Sector

Table 12: IonQ End Users by Industry (continued)

Sector	End Users	Description and Example
AI	IonQ	Develop quantum-classical deep learning architecture for large language model fine-tuning, leading to an overall improvement in prediction accuracy over a comparable classical baseline (Source) .
Physics	University of Washington	Quantum solutions to model fundamental physics processes and understand why the universe is composed predominantly of matter rather than antimatter. IonQ's technique allows scientists simulate the nuclear dynamics on the shortest of time-scales (10^{-24} seconds) (Source) .

Quantinuum

Quantinuum expects that, by the end of the decade, the accelerated hardware roadmap will deliver a fully fault-tolerant and universal quantum computer capable of executing millions of operations on hundreds of logical qubits.
Quantinuum has set a path to their fifth-generation system, Quantinuum Apollo, a machine that delivers scientific advantage and a commercial tipping point this decade.

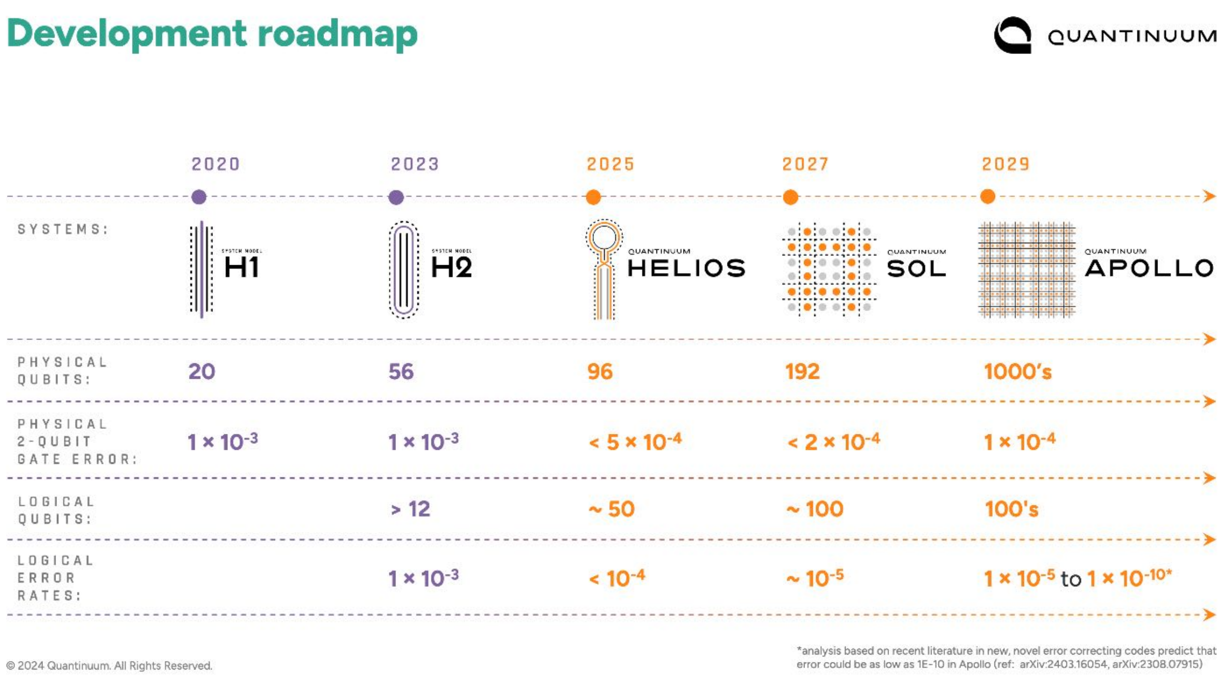


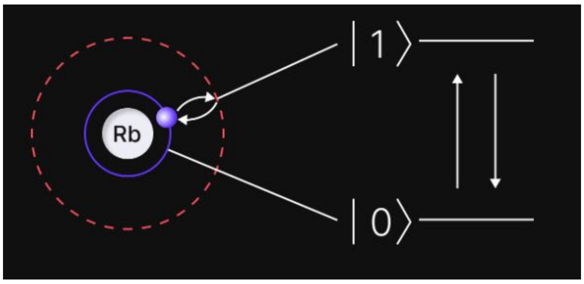
Figure 45: 2025 Development Roadmap, Quantinuum.

Sector	End Users	Description and Example
Cybersecurity	Thales & JPMorgan	Develop a quantum-safe commercial solution for secure key generation, management, and protection by generating highly unpredictable keys with quantum-computing strengthened entropy (Source) .
Research	Princeton & NIST	Delivered a crucial result in Quantum Error Correction (QEC) by using “concatenated codes” to exponentially suppress noise. Their results, now widely referred to as the “threshold theorem”, provide realistic possibility of fault-tolerant quantum computing (Source) .
Bank	HSBC	Use quantum computing-hardened cryptographic keys to mitigate cyber threats & Explore quantum machine learning (QML) for HSBC’s business, with fraud detection as a priority (Source).
AI	Quixer (platform)	Quixer- Quantinuum’s quantum transformer in Natural Language Processing. It is tailored for quantum circuits and thus more resource efficient than most competing approaches (Source) .
Physics	DESY	The first project describes a quantum machine learning technique for untangling data from the LHC. More recently, Quantinuum System Model is used to tackle Lattice Gauge Theory (LGT), as it’s a favorite contender for quantum advantage in High Energy Physics (HEP) (Source) .
Research	Qpurpose (funded by Q-Access call)	Perform computations to probe interesting mathematical conjectures and develop new quantum algorithms and advance techniques for error mitigation and correction (Source) .

Table 13: Quantinuum End Users by Industry Sectors

7.2.3. Neutral Atoms

Using individual atoms trapped in tightly focused optical tweezers, it is possible to create large qubit registers in one [112], two [113] or three [114] dimensions, with arrays of over 1000 sites [115]. Qubits are encoded in the hyperfine-ground states for the alkali atoms, or on optical clock transitions with alkaline-earth species, offering long coherence times of up to 40s [116].



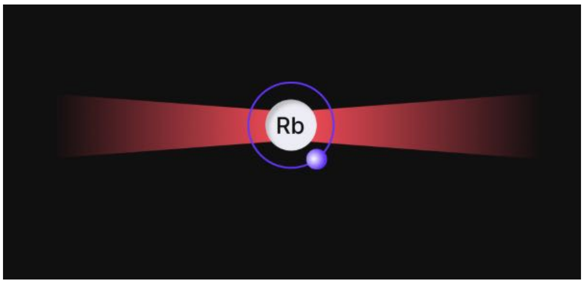


Figure 46: Left: two distinct energy level of netural atoms to represent '0' and '1' to form a qubit. Right: lasers suppress atomic movement, leading to huge coherence times.

Single qubit gates can be implemented using microwave or optical fields [117]. In addition, atoms are excited to high quantum number Rydberg states and form an effect known as Rydberg blockade. This effect allows to implement high-fidelity two or three qubit gate operations [118–120].

Leading quantum hardware providers and their roadmaps

QuEra

QuEra is based in Boston and built on pioneering research from nearby Harvard University and MIT. Today it operates the world's largest publicly accessible quantum computer, available over a major public cloud and for on-premises delivery.
QuEra's technology enables a clear roadmap to millions of qubits. Users can access QuEra's first-generation machine, Aquila, a 256-qubit Quantum Processor on Amazon Braket.

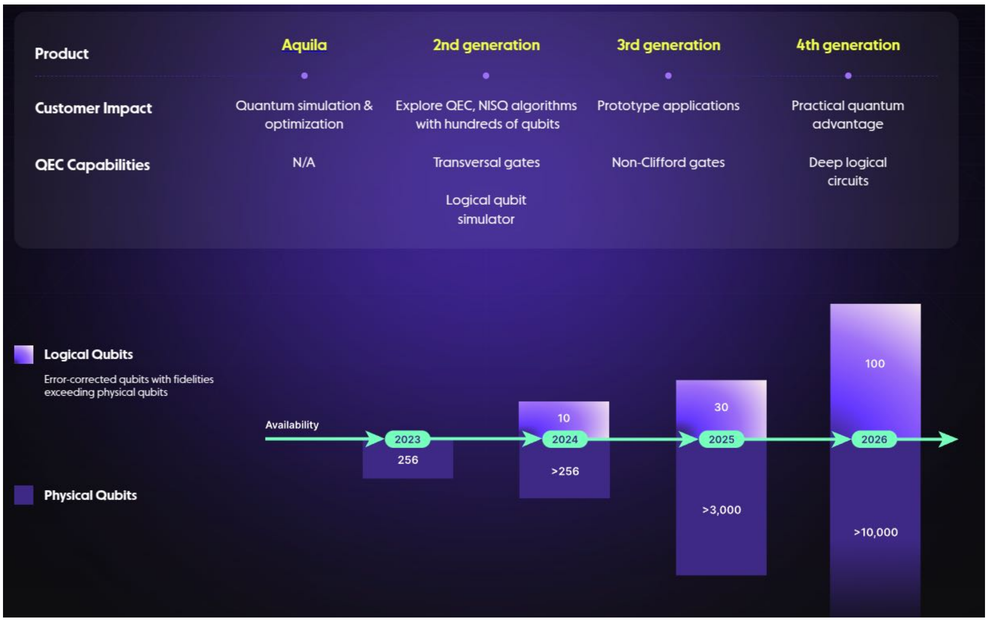


Figure 47: 2025 Development Roadmap, QuEra.

Sector	End User / Project	Description
Hardware	Google Quantum AI 🇺🇸	Accelerate technology advancements in the neutral atom space to enhance quantum error correction capabilities. Expand Google's quantum portfolio beyond its primary work in superconducting qubits (Source) .
Weather	Moody's 🇺🇸	Build a quantum reservoir computing-based model, capable of processing climate model outputs and storm environment parameters to provide more accurate tropical storm forecasting (Source) .
Medicine	Merck KGaA 🇩🇪	Quantum Reservoir Computing (QRC) to predict pharmacological activity using molecular fingerprints. QRC provides a more interpretable model with less training data than its classical counterpart (Source) .
Telecommunication	Cinfo 🇨🇭 & Kipu Quantum 🇩🇪	Quantum optimization for a complex telecommunication network in Spain to encounter equipment failures, natural disasters, or geopolitical issues. It quantifies the network's resilience and redundancy via the Maximum Independent Set (MIS). (Source)

Table 14: QuEra Use Cases by Sector

Infleqtion

Infleqtion is built on 15 years of pioneering quantum research from ColdQuanta to construct quantum computers, precision sensors, and quantum software for governments, enterprises, and research institutions. Their commercial portfolio also includes quantum Radio Frequency (RF) systems (world's first quantum RF trials and first RF sensor deployed) , quantum clocks, and inertial navigation solutions. Its scalable and versatile quantum technology is used by organizations around the globe and deployed by NASA on the International Space Station.

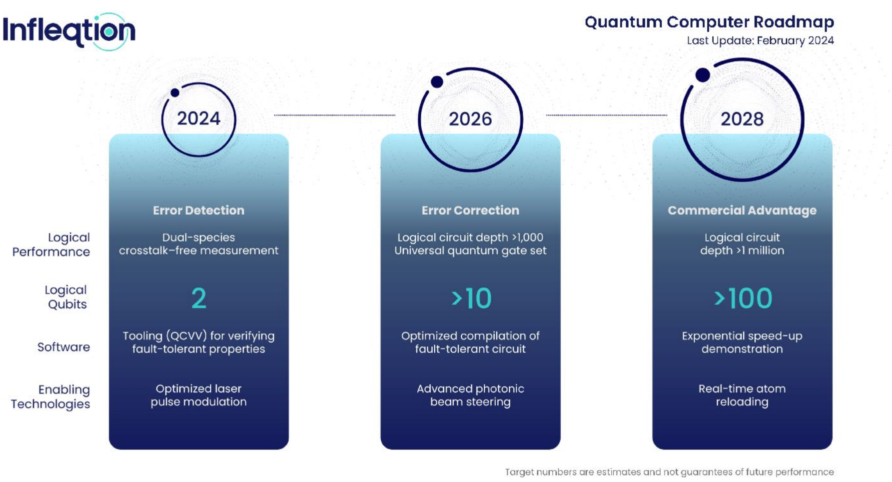


Figure 48: 2024 Development Roadmap, Infleqtion.

Sector	End User / Project	Description
Aerospace	NASA	Developed Physics Package Assembly for: 1) NASA's Cold Atom Lab aboard the International Space Station that allow deeper study of quantum behavior and enhanced precision measurement; 2) JPL Quantum Gravity Gradiometer Pathfinder to measure Earth's gravity field from space (Source) .
Energy	U.S. Department of Energy	Develop quantum-enhanced solutions for energy grid optimization to: 1) improve energy demand forecasting using quantum contexual machine learning; 2) optimize phasor measurement unit placement for better grid observability; 3) enhance nuclear energy modeling at the subatomic level (Source) .
Sensor	US Navy	Contextual Machine Learning (CML) with Quantum-Inspired Algorithms to sensor data streams. It enhances real-time RF signal processing, improving situational awareness, security, and operational efficiency in both current RF systems and next-generation Quantum RF sensors (Source) .
Medicine	University of Chicago and MIT	Develop quantum cancer biomarker identification algorithms, which integrates diverse data modalities, including genomics, transcriptomics, and pathology to drive personalized diagnostics and treatments in the clinical cancer care (Source) .
Finance	JP Morgan Chase	Quantum-classical hybrid methods to improve portfolio construction for higher risk-adjusted returns — Sharpe ratio of 0.99 vs. 0.88 for traditional equal-weighted portfolios and greater efficiency — near-optimal portfolios found with far fewer samples (Source) .

Table 15: Infleqtion Use Cases by Sector

Pasqal

PasqaL is founded in 2019 as a spinoff from Institut d'Optique in Paris, which emerged from groundbreaking research in neutral atom quantum computing led by Nobel prized quantum physics pioneer Alain Aspect. Today, Pasqal stands at the forefront of quantum innovation, partnering with industry leaders across finance, energy, and manufacturing to solve complex computational challenges previously thought impossible with classical computing systems.

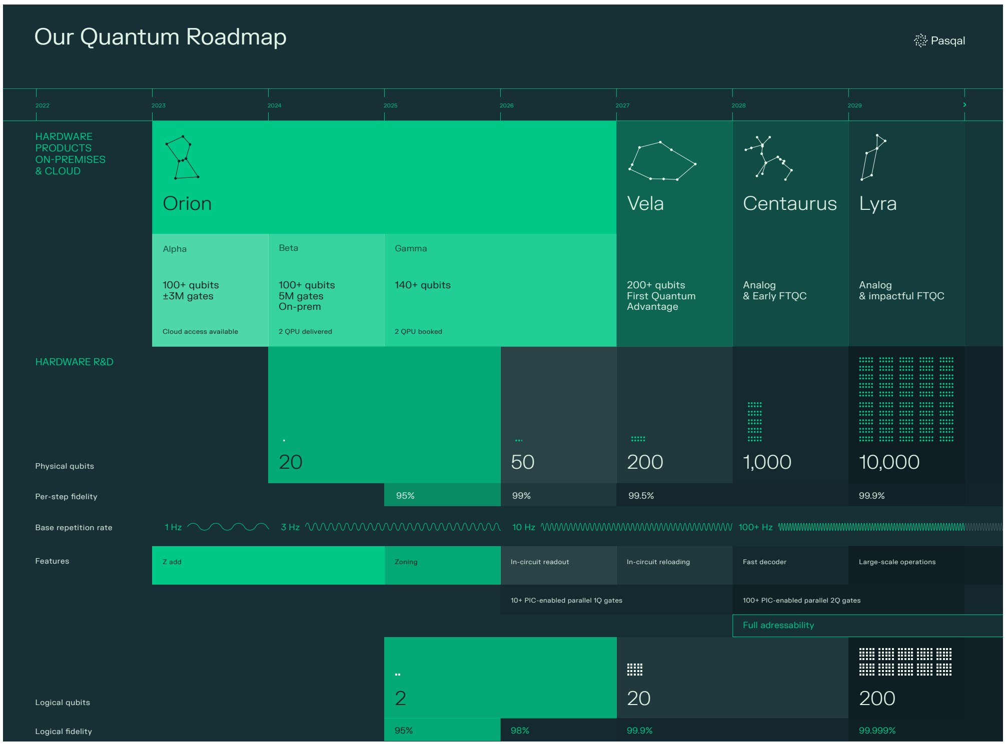


Figure 49: 2025 Development Roadmap, Pasqal.

Sector	End User / Project	Description
Aerospace & Defense	Thales 🇺🇸	Model the satellite scheduling problem as a Maximum Independent Set (MIS) problem to seek quantum-centric approach to the optimization of satellite positioning and resource allocation under complex constraints, aiming to improve scheduling flexibility and reaction times (Source) .
Material	BMV group 🇩🇪	Develop a quantum solution to improve simulation of metal forming, predicting material behavior and optimizing manufacturing settings, with future use in structural and crash simulations. This will later support structural analysis and crash simulations (Source) .
Energy	Electricite de France (EDF) 🇫🇷	Quantum optimization for energy distribution and the environmental variables affecting wind farms and photovoltaic plants. Another project involves simulating the ageing of materials within nuclear power plants to ensure safety and operational integrity (Source) .

Table 16: Pasqal Use Cases by Sector

Sector	End Users	Description and Example
Bank	Credit Agricole CIB	Quantum computing's potential, e.g., a quantum-enhanced machine learning, in predicting credit rating downgrades 6 to 15 months in advance for financial risk management (Source) .
Weather & Farm	BASF	Solve complex nonlinear differential equations in physics-based weather models via quantum neural networks. It is aligned with BASF's digital farming product for optimized crop platform (Source).

Table 16: Pasqal Quantum End Users by Industry (continued)

7.2.4. Photonic

A photonic quantum computer uses photons to store and process quantum information. It provides an attractive alternative to other approaches as it offers the potential of room temperature operation and the promise of scalability.

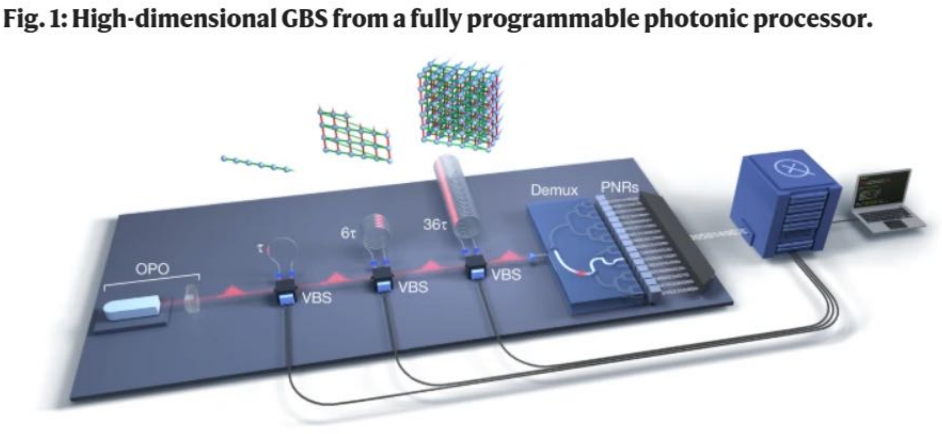


Figure 50: High-dimensional GBS from a fully programmable photonic processor - Xanadu.

There are several companies attempting to produce a universal quantum computing platform using photons, each one championing their own variants of the methodologies. Recent articles have claimed fidelities as high as 99.69% [121–123]. A commercial system has claimed quantum computational advantage [121] using both Gaussian boson Sampling (GBS) and squeezed states of light for differing modes to produce Gottesman, Kitaev and Preskill (GKP) qubits.

Leading quantum hardware providers and their roadmaps

ORCA Computing

ORCA Computing is a UK-based quantum computing company developing photonic quantum computers. The PT Series are special purpose photonic quantum computers for machine learning. Available in two models – PT-1 and PT-2 – both are designed to break the barriers to quantum adoption, as well as feasible integration into existing HPC infrastructure.

ORCA Computing: Current-Generation Photonic Systems

PT-1

- The exploration system
- Hybrid quantum–classical machine learning
- Simplified integration with existing ML
- Lowest entry costs

PT-2

- High-performance photonic computing
- Quantum-accelerated generative AI & optimisation
- Data centre & AI factory ready
- lowest total cost of ownership

Sector	End User / Project	Description
Energy	Frazer-Nash 🇬🇧	Quantum solution for "Unit Commitment Problem" to optimize grid balancing for the National Grid Electricity Supply Operator (ESO). The matching power generation to demand can prevent blackouts, which depends on different power sources (gas, solar, wind, nuclear, etc.) vary in cost, availability and response time (Source) .
Chemistry	bp 🇬🇧	Hybrid quantum-classical approach using generative adversarial network (GAN) algorithms to generate low-energy conformations of small to medium hydrocarbons, aiming to correctly predict the 3D shape, or conformations, of molecules (Source)
Medicine	DTU, Sparrow Quantum 🇩🇰 & PSNC 🇵🇱	Quantum enhanced vaccine design: capture biological relationships governing peptide-MHC (Major histocompatibility complex) presentation, and design peptides with a higher likelihood of triggering an immune response by being immunogenic (Source)
Routing	The Port of Dover 🇬🇧	Quantum Unconstrained Binary Optimization (QUBO) to enhance operational efficiency and reducing congestion of the port, cutting down queues and waiting times (Source)
Defense	UK Ministry of Defence 🇬🇧	Hybrid quantum-classical generative machine learning to the infilling and improving satellite images, which can be applied on removing airplane contrails from satellite images, improving sea and land surface temperature monitoring, generating super-resolution imagery from low-resolution images, forecasting near-term storm cloud trajectories and in-filling of algal bloom data (Source)
Manufacture	Arup 🇬🇧	Apply quantum computing in bridge design, ensuring a bridge is strong enough to support its load, without over-engineering and wasting resources. The bridge deck loading problem is a combinatorial optimization challenge, where the goal is to find the worst-case combination of vehicle positions on a grid covering the bridge deck. (Source)

Table 17: ORCA Computing Use Cases by Sector

7.3. Cost models and compared pricing plans for QPU resource

Commercial quantum computing access is currently based on two major pricing models:

Execution Time Model, typical factors include:

- wall-clock runtime,
- number of circuit repetitions (shots).

Formula-Based Model, typical factors include:

- the configuration of the quantum circuits
- number of measurements.

1. **Execution Time (HPC-like) Model**
   Users are charged per job submission or execution session. Pricing depends on the wall-clock runtime or the number of circuit repetitions (shots).

2. **Formula-Based (Resource-Weighted) Model**
   Here, the price is determined by a provider-specific formula that accounts for the resources consumed by the circuit. Two-qubit gates usually dominate the cost because they are the most technologically demanding operations.

Vendor	Free Resource	Pricing (Pay As You Go Plan)
IBM	10 minutes/month	$96/min
Rigetti	-	$0.02/millisecond
IQM	30 credits per month, based on availability	$0.30/second
OQC	-	Quote
Anyon Technologies	-	Quote
IonQ	$10,000 in free credits to qualified academics	$0.00022/1-qubit-gate, $0.000975/2-qubit-gate; Minimum price per program execution:$12.4166
Quantinuum	-	$15 / HQC unit
QuEra		Quote
Pasqal	Avaliable Soon	$15/hour on Emulator, $300 /hour on Quantum Hardware
Infleqtion		Quote
ORCA		Quote

Color Legend: Superconducting Trapped Ions Neutral Atoms Photonic

Advanced Subscription Plan

Beyond the basic *pay-as-you-go* plan, most quantum hardware providers also offer premium monthly subscription packages. These typically involve a fixed minimum monthly cost and may unlock advanced features such as priority job scheduling, enhanced access to hardware backends, and dedicated technical support from the provider.
Researchers and organizations interested in subscription-based access are encouraged to contact the corresponding hardware providers directly to tailor to their needs.

8. EuroQHPC Quantum-Classical Integration

EuroHPC quantum computers are designed to complement Europe's existing supercomputing infrastructure, significantly enhancing capabilities in scientific research, innovation, and industrial applications. Up to now, the EuroHPC JU has procured eight quantum computers.

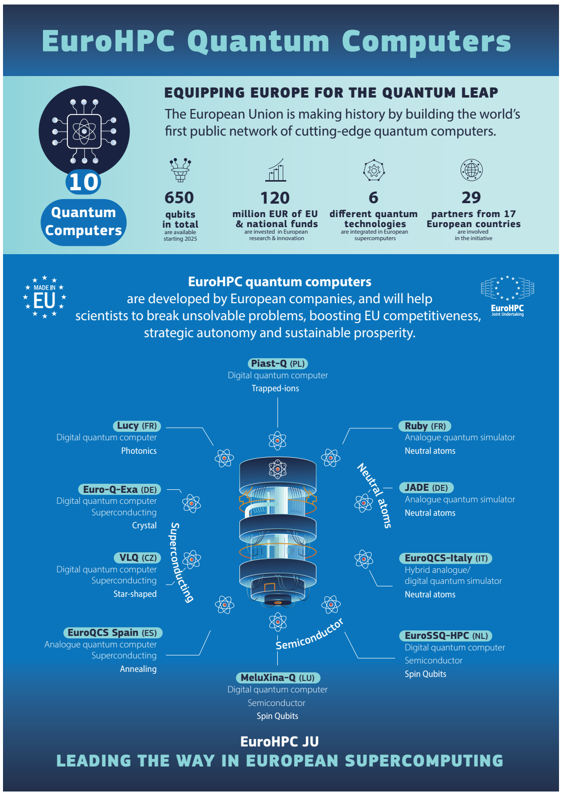


Figure 51: EuroHPC Quantum Computers. Source: The European High Performance Computing Joint Undertaking (EuroHPC JU) Project

8.1. Accessible quantum computers

JADE (Germany) A neutral-atom quantum simulator that arranges individual atoms with optical tweezers. It targets scalable simulations and hybrid HPC+quantum workflows in chemistry, materials, and optimization.

Ruby (France) A neutral-atom system similar to JADE, designed for accessible quantum simulation and application pilots across logistics, energy, and finance, tightly coupled to HPC resources.

Lucy (France) A photonic quantum computer, allowing operation at room temperature and natural compatibility with optical networking. It is positioned for hybrid HPC workloads and algorithm prototyping.

PIAST-Q (Poland) A trapped-ion quantum computer offering very long coherence times and all-to-all qubit connectivity, which simplifies algorithm mapping and enables deeper circuits than many other platform of similar size.

VLQ (Czechia) A superconducting-qubit processor engineered for efficient connectivity ("star-like" coupling) to reduce routing overhead. It supports gate-model algorithms and integration with HPC systems for pre/post-processing.

Euro-Q-Exa (Germany) A superconducting system delivered in two stages—an initial ∼54-qubit device followed by ∼150 qubits—to enable progressively more complex quantum algorithms within LRZ's HPC environment.

EuroQCS-Italy (Italy) A neutral-atom quantum *simulator* delivered in two phases: an analogue system with at least 140 qubits, then an upgrade to hybrid analogue/digital operation to broaden algorithmic reach and scientific use cases.

EuroQCS-Spain (Spain) A superconducting *quantum annealer* aimed at optimization problems common in ML, finance, and logistics, integrated with national supercomputing for data handling and workflow orchestration.

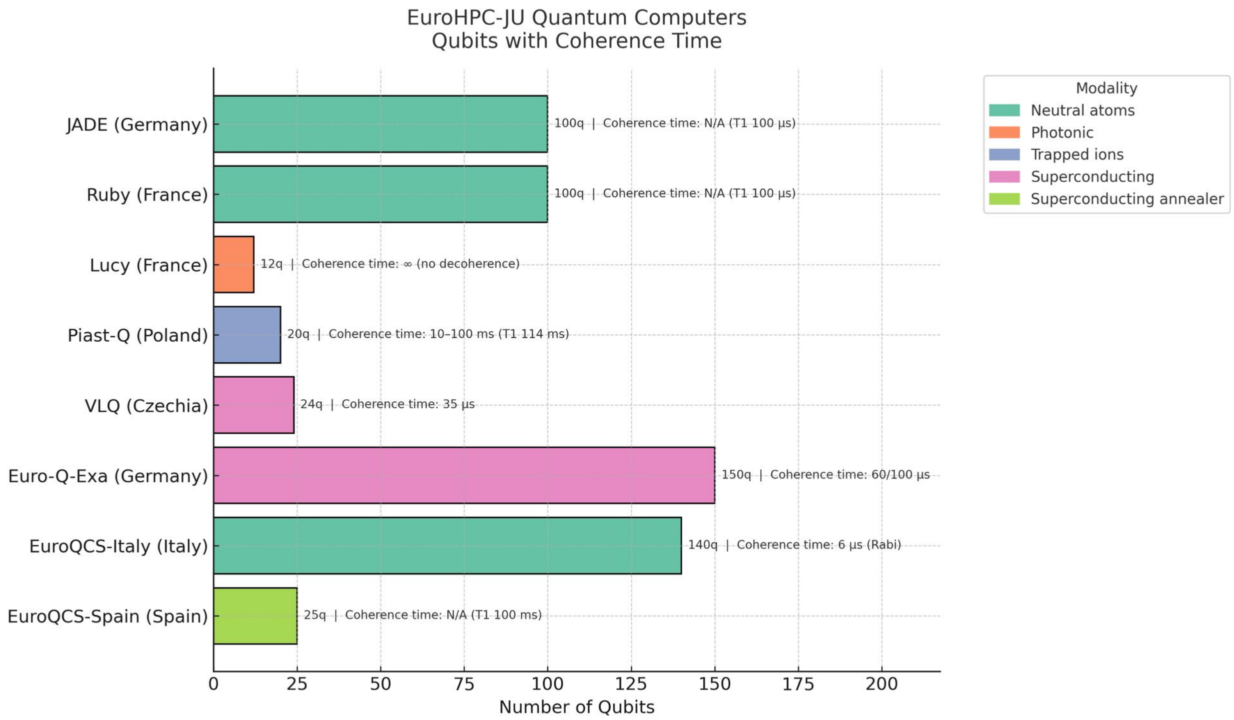


Figure 52: According to the official EuroHPC-JU descriptions, all listed quantum computers achieve a two-qubit gate fidelity exceeding 99%.

8.2. How to apply - current status of access

DeiC has submitted the question regarding how to apply HPC (and potential quantum integration resource) under EuroHPC JU with DeiC's representative in the Ministry, who also serves as a member of the EuroHPC board. It is anticipated that further clarification will be provided soon.

At present, the general expectation is that these resources will be made available through calls, administered in a manner similar to existing HPC resources. The first calls are expected to be announced during the fall. Below, we summarize the quantum computers in the project that provide early access now.

PIAST-Q Quantum Computer

System at a glance.

- **Platform:** Laser-based *trapped-ion* quantum computer.
- **Host / Operator:** Poznań Supercomputing and Networking Center (PCSS).
- **Supplier:** Alpine Quantum Technologies (AQT), Innsbruck.
- **HPC integration:** First coupled with the *ALTAIR* supercomputer; later with *PIAST-AI*, enabling hybrid classical–quantum workflows.

Early Access:

Please fill out the form at the bottom of this page to provide more information about your experiment and apply for the early access for the PIAST-Q system.

Use cases and scope.

- Hybrid applications in *quantum optimisation*, *chemistry*, *risk analysis*, and *machine learning*.
- Part of a broader EuroHPC effort to deploy multiple quantum modalities and integrate them with European HPC.

9. Quantum Interface with AI

9.1. The Remarkable Achievements and Application of AI

Artificial Intelligence (AI) and in particular Machine Learning (ML), has deeply embedded in nearly every aspect of modern society, driving innovation in science, technology, and daily life. In particular, the year 2024 was a landmark moment for recognition: breakthroughs in AI were honored with the *Nobel Prize in Physics* and the *Nobel Prize in Chemistry*, underpinning its revolutionary impact.

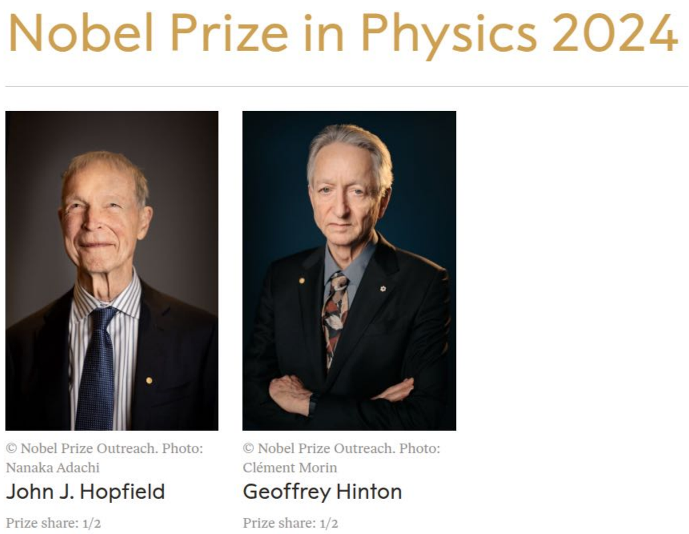


Figure 53: The Nobel Prize in Physics 2024 was awarded jointly to John J. Hopfield and Geoffrey Hinton "for foundational discoveries and inventions that enable machine learning with artificial neural networks". Source: NobelPrize.org.

Figure 54: The 2024 Nobel Prize in Chemistry was awarded to Google DeepMind researchers Demis Hassabis and John Jumper for developing AI programs that predict protein 3D structures, and to David Baker (University of Washington) for using AI to design entirely new proteins. Source: NobelPrize.org.

AI, ML, and Neural Networks, what are they?

Before exploring in detail how quantum technologies can enhance AI tasks, it is helpful to clarify the terminology first. The terms *Artificial Intelligence (AI)*, *Machine Learning (ML)*, and *Neural Networks (NN)* are often used interchangeably, but they refer to distinct concepts. Making these distinctions clear is especially important for non-specialist audiences.

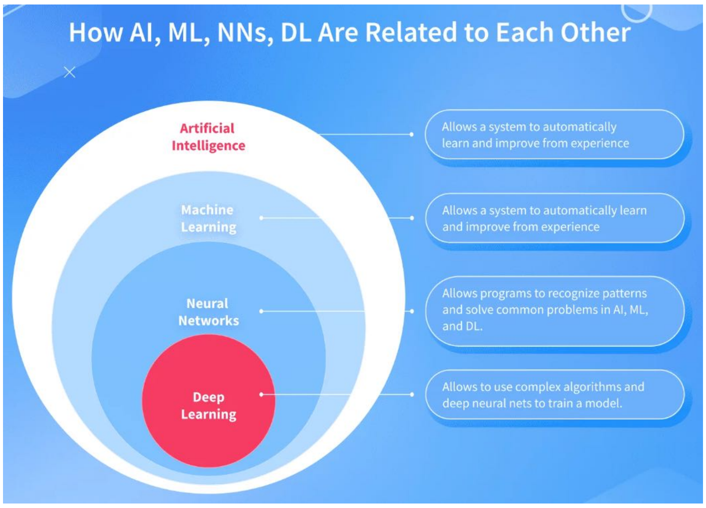


Figure 55: Relationship between AI, Machine Learning, Deep Learning, and Neural Networks. Source: Max Ushchenko, Head of Data and AI Practice at NIX.

Artificial Intelligence (AI)
AI is the *umbrella* of the three terms, referring to any technology that mimics human intelligence or cognitive functions such as reasoning, learning, and problem-solving. AI applications include natural language processing (e.g., Siri, Alexa).

Machine Learning (ML)
ML is a *subfield of AI* that focuses on algorithms that learn patterns from data, rather than being explicitly told with rules. ML techniques are now indispensable in scientific research, enabling powerful predictive models and data-driven discoveries.

Neural Networks (NN)
NNs are a *specific family of ML algorithms* inspired by the structure and function of biological neurons. Through training on large datasets, neural networks learn to adjust their weights on the edges that connect each node (artificial neuron) to improve accuracy. Once trained, they become powerful tools for fast classification and pattern recognition.

How to quantify the development of broader Artificial General Intelligence

The broader field of *Artificial General Intelligence* (AGI) aims at the development of machines capable of human-level reasoning and flexible problem-solving; this area often involves different challenges than those addressed by current quantum computing research. Recently, Institute for Human-Centered AI, Stanford University has published a comprehensive review - The AI Index 2025 Annual Report to trace the most critical trends shaping the field. In a domain advancing at breakneck speed, the Index provides essential context—helping us understand where AI stands today, how it got here, and where it may be headed next.

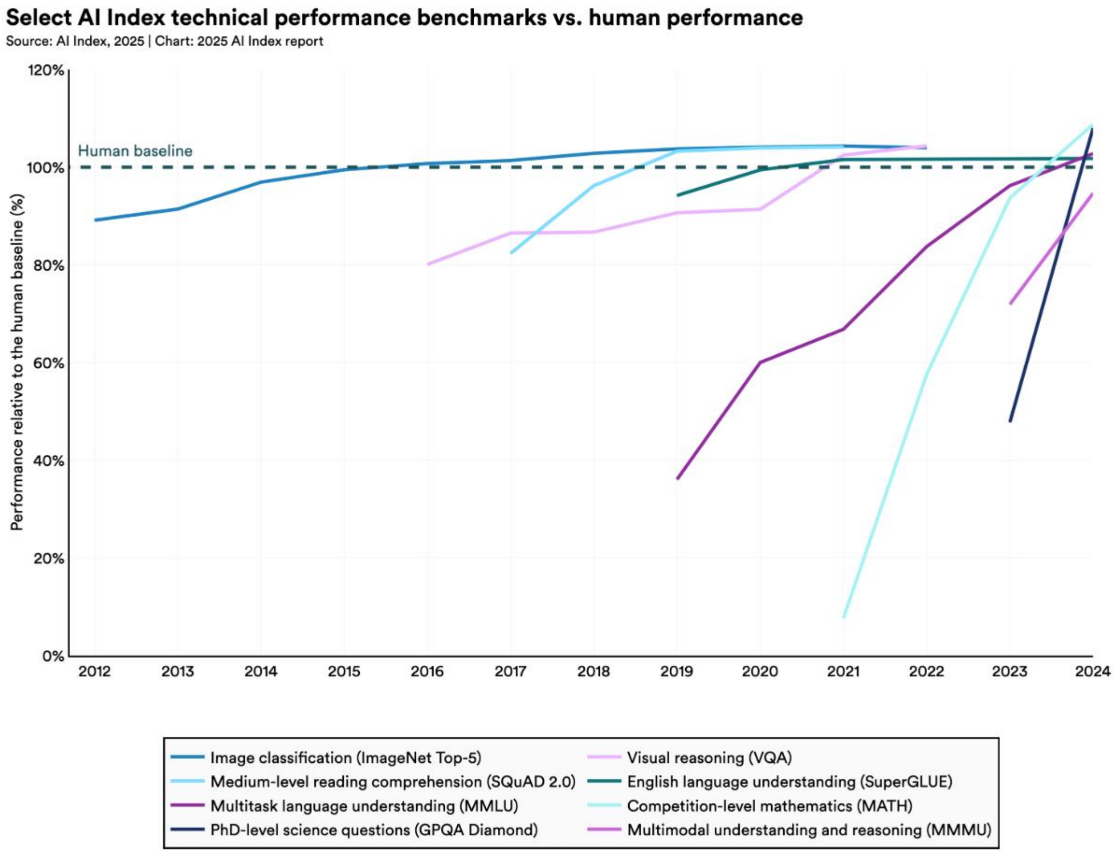


Figure 56: Benchmarks have become a central role to quantify AI capabilities and how they are advancing so quickly. In 2023, researchers introduced new benchmarks—MMMU, GPQA, and SWE-bench—to test the limits of advanced AI systems. Just a year later, performance sharply increased: scores rose by 18.8, 48.9, and 67.3 percentage points respectively. Source: The AI Index 2025 Annual Report by Stanford University.

9.2. Why seek quantum advantage in Machine Learning

In contrast, ML represents a mathematically well-defined subfield of AI, characterized by mature theoretical foundations and a wide range of real-world applications. When discussing the search for *quantum advantage*, quantum machine learning naturally emerges as a promising target, since it focuses on clearly formulated computational and data-driven tasks where quantum resources can potentially deliver demonstrable algorithmic improvements.

Main Task of Machine Learning — Finding Patterns in Data

Machine Learning is a *subset of Artificial Intelligence (AI)* whose core task is to *find patterns from data* rather than being explicitly revealed with every rule. From early statistical methods to modern computational techniques, this quest has continually driven the development of new tools and algorithms.

For example, long before the invention of digital computers, human beings strove to analyze observational data. One of the most known applied regimes falls into astronomy; here, scholars used celestial records to infer models and laws. This tradition of pattern analysis laid the foundations for key mathematical innovations during the Enlightenment, including methods for solving linear equations (Newton-Gauss), learning optima via gradient descent (Newton), polynomial interpolation

(Lagrange), and least-squares fitting (Laplace) [129].

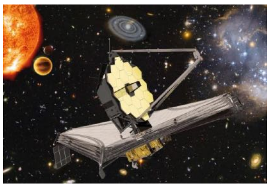
	Who?		How?	
Ancient Rome	Ptolemy		Propose a geocentric model of the cosmos, with complex epicycles to explain the retrograde motions of the planets	**Visual Perception**
Renaissance	Kepler		Analyze the data of Copernicus and Brahe to reveal that planets move in ellipses with the Sun at one focus of the ellipse. Give rise to mathematical techniques, e.g., for solving linear equations (Newton–Gauss).	**Math Formulation**
Modern	James Webb Telescope		Use machine learning algorithms to examine the treasure trove of data from the James Webb Space Telescope, changing our understanding of the universe	**Machine Learning**

Figure 57: Historical example: extracting patterns from astronomical observations.

In the modern era, this pursuit has naturally led us to the widely used *machine learning (ML)* methods, particularly deep neural networks, to detect intricate and subtle patterns in large-scale datasets, as well as allowing us to simulate sophisticated patterns at unprecedented scales.

Focus of This Chapter: Quantum Machine Learning

Quantum Machine Learning (QML) investigates how uniquely quantum features, such as superposition, entanglement, and interference, can be leveraged to enhance core ML tasks including pattern recognition, optimization, and generative modeling.

Why many expect quantum to enhance classical ML

> Quantum mechanics is well known to produce atypical patterns in data. Classical machine learning methods such as deep neural networks frequently have the feature that they can both recognize statistical patterns in data and produce data that possess the same statistical patterns: they recognize the patterns that they produce.
> This observation suggests the following hope. If small quantum information processors can produce statistical patterns that are computationally difficult for a classical computer to produce, then perhaps they can also recognize patterns that are equally difficult to recognize classically.
>
> — Quantum Machine Learning *by Biamonte et al. [129]*

The integration of quantum components can be both interpreted broadly as inspiring new ML methods via quantum theory, using ML to analyze quantum experiments, or more narrowly as machine learning with or assisted by quantum computers [124]. The remainder of this chapter concentrates on the later, namely on how **quantum ingredients can enhance machine learning**, both in the current *NISQ* era and in the future *FTQC* period. The structure and content of this chapter are inspired by the excellent review article *Quantum Machine Learning* by Biamonte *et al.* [129] and [124].

Benchmark of quantum enhancement in machine learning

Often, the field asks whether quantum computers can improve learning tasks such as pattern recognition or optimization compared to classical systems. In fact, the answer relies on a comprehensive comparison based on a collection of benchmarks as illustrated below [124].

Benchmarking a "Better" ML Model (Classical or Quantum)

Asymptotic runtime

Growth of computational cost with input size n (e.g., $O(n)$ vs. $O(n^2)$). *Quantum goal:* provable slower growth of runtime based on quantum speedup or empirical improvement.

Energy efficiency

Energy consumption per training task. *Quantum strength:* at large scales, quantum devices may exhibit more energy efficiency as depicted in [130].

Accuracy / loss

Task specific quality (e.g., confusion matrix, RMSE, log-loss). *Quantum goal:* better evaluation metrics.

Generalization

A model generalizes well if it maintains high accuracy on unseen data

Training time

counting of iterations for the error/loss to converge. *Quantum goal:* faster convergence with fewer epochs.

Expressivity

Capacity represent or approximate different kinds of patterns in data - a measure of the richness or flexibility of the model's hypothesis space. *Quantum goal:* leverage entanglement to capture structure with fewer parameters.

Model Scalability

Can increase model size (number of training parameters) without instability *Quantum strength:* Quantum circuits can represent highly entangled functions with far fewer trainable parameters

Expectation for QML: Aim for improvements in *multiple* criteria, not only speed.

How quantum hardware advancement can impact ML

Meanwhile, machine learning tasks can benefit from the advancement of quantum hardware via:

1. **Scaling of logical qubits:** The hardware roadmaps indicate that the community is transitioning from today's enforced hybrid classical–quantum models, limited by available resources, toward pure quantum architectures capable of hosting entire learning pipelines. These new degrees of freedom enable more components of classical ML models—such as data encoding, training structures, and readout—to be replaced with quantum versions.

2. **Improved fidelities and coherence:** Higher gate fidelities and longer coherence times make it realistic to explore variational circuits with depth exceeding 100 layers. This scale is sufficient

to represent multi-layer quantum neural networks (QNNs) and to fully exploit gradient-based optimizers.

Outlook

Looking ahead, with the rapid development of both AI and quantum technologies, it will be fascinating to explore how increasingly powerful quantum devices, particularly in the future *FTQC* era, might contribute to advancing the ambitious goals of *Artificial General Intelligence (AGI)*.

9.3. (Hybrid) Quantum Machine Learning - NISQ era

Here, we explore how quantum techniques can be integrated into conventional machine learning methods, examining their potential to enhance training performance, scalability, and expressive power. These classical methods are not only well understood and widely deployed on classical hardware, but they also provide natural entry points for incorporating quantum components in the *NISQ* era.

Unlike fault-tolerant quantum computers, current quantum devices are constrained by factors such as noise, limited coherence times, and restricted qubit connectivity. Nevertheless, the quantum devices in the current generation do in principle have the power to test the advantages of quantum computing, but the constraints have a profound impact on quantum algorithmic design. The limited quantum resource enforces users to strategically identify and replace selected segments of the classical machine learning pipeline with quantum subroutines, thereby creating **hybrid QML algorithms**, a typical model is illustrated in Figure [58].

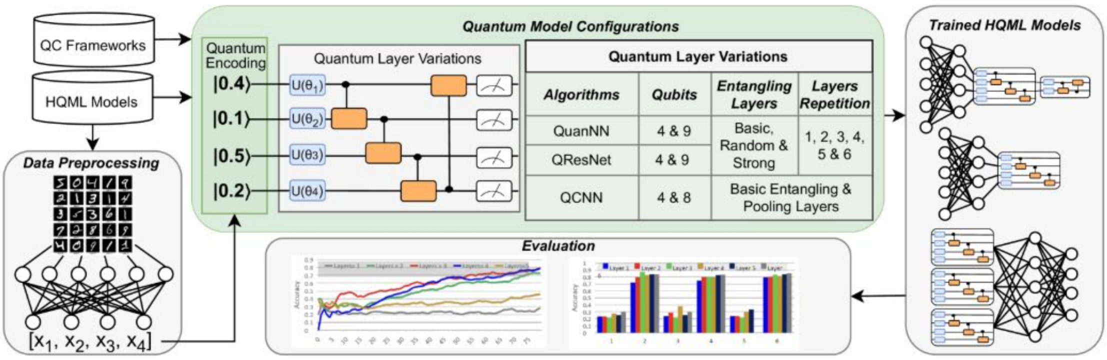


Figure 58: Pipeline of hybrid QML algorithms, the green part indicates the insertion of quantum component. [125]

9.3.1. Quantum Neural Network and Quantum Kernel Methods

The interaction of quantum computing and neural networks has increasingly converged on using **variational quantum circuits (VQCs)** to learn data representations, rather than constructing direct quantum analogues of classical neural networks. A typical QNN involves three key components:

- **Data Encoding (Feature Mapping)**: Classical data $\mathbf{x}$ are first embedded into a quantum state $|\phi(\mathbf{x})\rangle$ using an encoding unitary $U_{\phi(\mathbf{x})}$. This can be achieved through various schemes such as:
  - *Angle encoding*: classical features modulate single-qubit rotation angles (e.g., $R_y(x_i)$).

- *Basis encoding*: binary strings map directly to computational basis states.
- *Amplitude encoding*: normalized data vectors define the amplitude distribution of a multi-qubit state.

The choice of encoding determines the effective feature space in which the QNN operates.

- **Parameterized Quantum Gates (Variational Layers)**: The encoded quantum state is processed through a sequence of parameterized unitary operations $U_{\boldsymbol{\theta}}$. These gates define the trainable parameters $\boldsymbol{\theta}$ as well as how quantum entanglement are formed across qubits.
- **Measurements (Readout and Loss Evaluation)**: After the parameterized evolution, specific observables O are measured to extract classical information, e.g., a classical loss function $\mathcal{L}(\boldsymbol{\theta})$, which guides parameter updates through optimization techniques such as the parameter-shift rule or gradient-free methods.

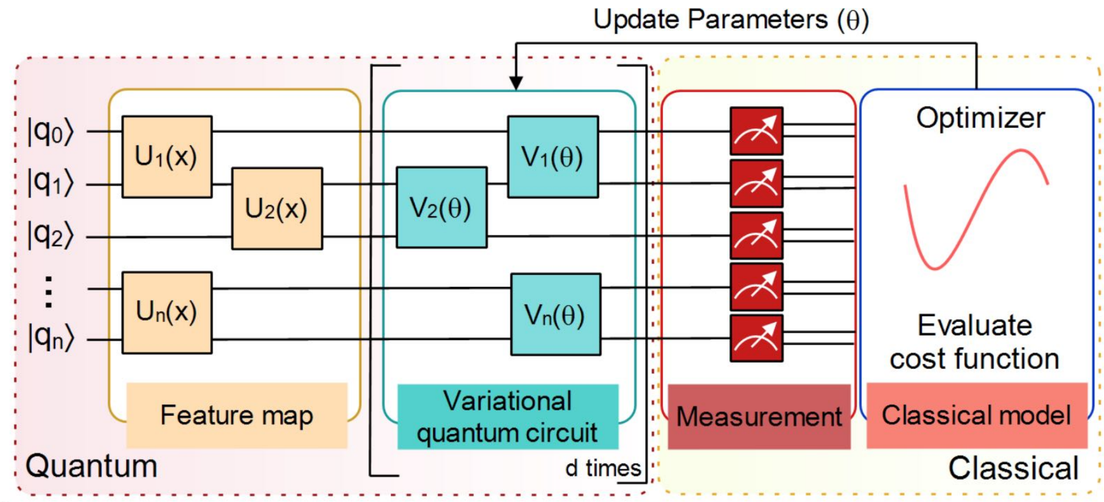


Figure 59: Schematic representation of a variational quantum circuit (VQC). These quantum components are coupled with a classical optimization routine, forming an iterative hybrid quantum–classical learning loop [131]

.

As a result, quantum integration can expand to many popular classical NN architectures, such as:

- **Quantum Convolutional Neural Networks (QCNNs)**: use quantum filters and pooling operations to extract multi-scale features, making them particularly suitable for tasks such as quantum state classification and phase recognition in condensed matter systems.
- **Quantum Convolutional Neural Networks (QCNNs)**: graphs are encoded into quantum state, aiming to efficiently represent and learn graph structured data, such as molecular structures or network topologies, within the Hilbert space.
- **Quantum Variational Autoencoders (QVAEs)**: the latent space is represented by quantum states, enabling compact data representations and the generation of complex distributions that are difficult to model classically.
- **Quantum Generative Adversarial Networks (QGAN)**: extend the GAN framework by implementing the generator, discriminator, or both as VQCs, aiming to learn and reproduce complex data structure more efficiently than classical GANs.

Each of these models leverages parameterized quantum circuits to reproduce the core functionalities of their classical counterparts while exploiting quantum effects such as superposition and entanglement to achieve potentially superior expressivity and representation power.

A use case of quantum neural network

The published work "Molecular design with automated quantum computing-based deep learning and optimization" (npj Computational Materials, 2023) [132] presents a hybrid quantum–classical framework for molecular property prediction and inverse molecular design. The authors combine graph-based molecular representations, deep learning, and quantum computing to accelerate the discovery of molecules with desired chemical properties.

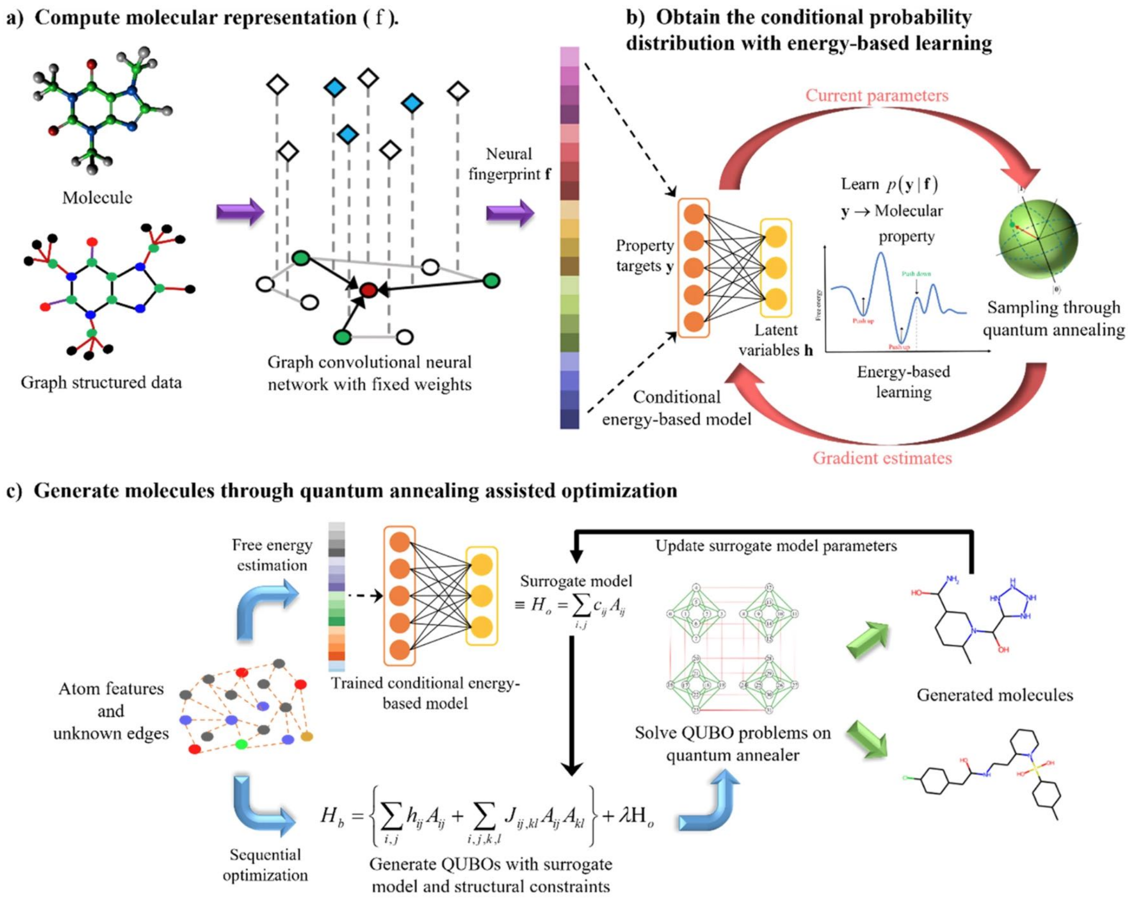


Figure 60: Schematic description of automated quantum computing-based deep learning and optimization to generate molecular target [132]

.

The method begins by encoding molecules as graphs and extracting neural fingerprints using a graph convolutional network. These fingerprints are then used in a conditional energy-based model that predicts molecular properties. Instead of relying solely on classical training, the model uses quantum annealing to perform sampling and optimization within the energy landscape, effectively integrating a quantum-assisted learning component.

For the inverse design task, the authors reformulate molecular generation as a quadratic unconstrained binary optimization (QUBO) problem. This allows the use of a quantum annealer to search efficiently for molecular structures that satisfy target property constraints. The framework iteratively refines the generated molecules through quantum sampling and classical optimization.

Results on benchmark datasets (e.g., ZINC) show that the quantum-assisted model improves both prediction accuracy and molecular validity compared to purely classical baselines. It also demonstrates faster convergence in subproblems and better exploration of chemical space. The study highlights that even with today's noisy quantum hardware, hybrid quantum-classical systems can offer practical advantages in molecular design and optimization.

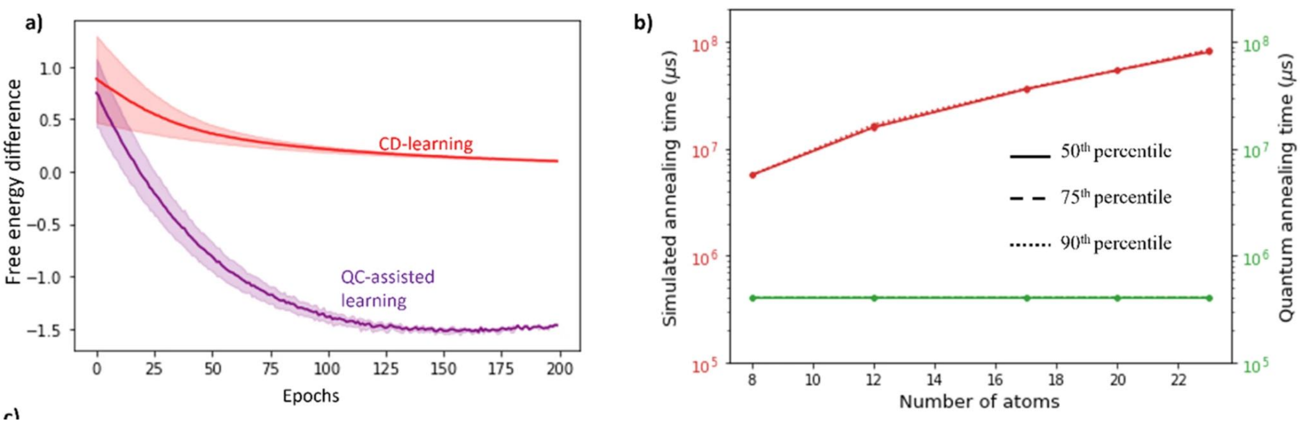


Figure 61: Performance comparisons between quantum and classical approaches.

Quantum support vector machines and kernel methods

Kernel methods [134] solve non-linear problems by finding an optimal separating hyperplane, such that all training examples of one class are found only on one side of the hyperplane. In particular, it implicitly maps input data $\mathbf{x} \in \mathcal{X}$ into a high-dimensional feature space $\mathcal{F}$ through a mapping $\phi : \mathcal{X} \to \mathcal{F}$. Instead of computing $\phi(\mathbf{x})$ explicitly, kernel methods rely on the *kernel function*

$$k(\mathbf{x}_i, \mathbf{x}_j) = \langle \phi(\mathbf{x}_i), \phi(\mathbf{x}_j) \rangle_{\mathcal{F}},$$

which measures the similarity between data points. This allows efficient training of models such as the *Support Vector Machine (SVM)* without incurring the computational cost of explicit feature-space evaluation.

Quantum advantage:

Quantum kernel methods exploit the exponential expressivity of Hilbert spaces spanned by quantum states. A *quantum feature map* encodes classical data $\mathbf{x}$ into a quantum state $|\phi(\mathbf{x})\rangle$, such that the kernel is defined as

$$k_Q(\mathbf{x}_i, \mathbf{x}_j) = |\langle \phi(\mathbf{x}_i) | \phi(\mathbf{x}_j) \rangle|^2,$$

which can be estimated efficiently on a quantum computer.

Moreover, a quantum support vector machine (QSVM) can also leverage the full potential of quantum Basic Linear Algebra Subroutines (qBLAS) — most notably through the Harrow–Hassidim–Lloyd (HHL) algorithm, which is demonstrated in Section [9.4]. All operations required to construct the optimal separating hyperplane, as well as to determine on which side of the hyperplane a test vector lies, can in principle be executed in time polynomial in $\log N$, where N denotes the dimension of the matrix used to prepare the quantum representation of the hyperplane vector. For example, this approach has been experimentally demonstrated on a nuclear magnetic resonance (NMR) quantum testbed for a handwritten digit recognition task [135].

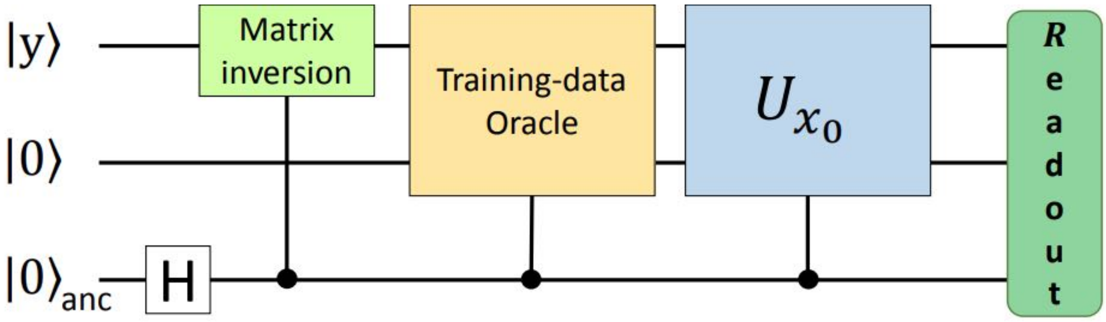


FIG. 2. (Color online) The schematic diagram of the quantum SVM. An ancillary qubit is added here to readout the classification result. The auxiliary registers for matrix inversion are not shown here.

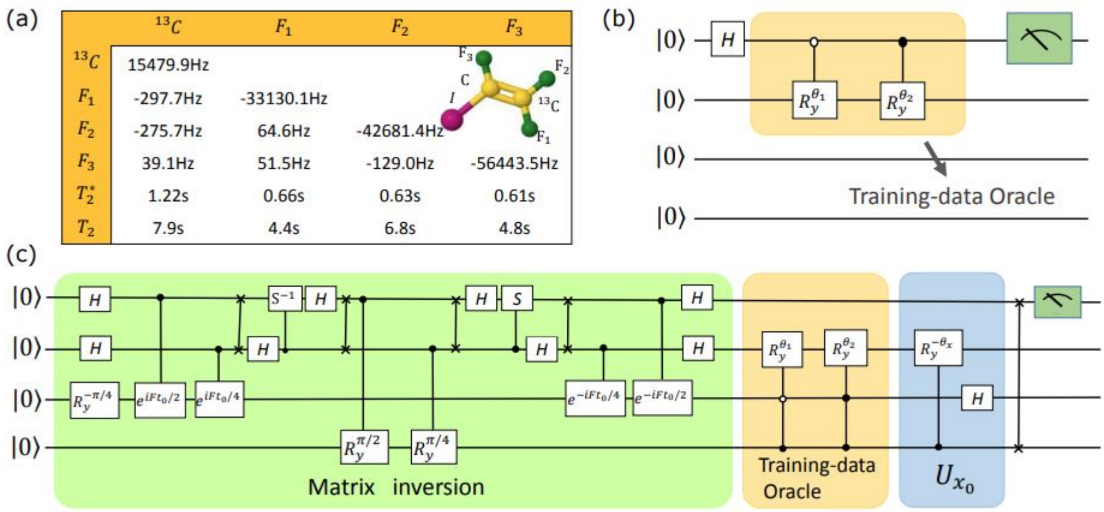


Figure 62: Schematic workflow of quantum support vector machine for a handwritten digit recognition task [135]

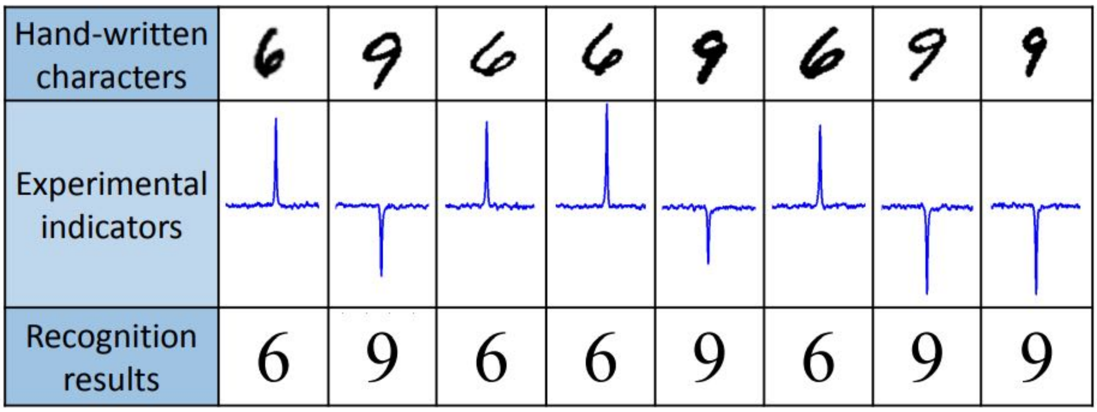


Figure 63: The recognition results are indicated by the orientation of the labeled peak in ^{13}C spectrum, which classifies the incoming hand-written character correspondingly [135]

9.3.2. Other interesting quantum machine learning methods

Quantum Neural Networks (QNNs) and Quantum Kernel methods form the backbone of many modern quantum machine learning approaches. Meanwhile, other quantum learning methods, such as quantum principal component analysis, quantum transformer, are tailored for more specific applications. For completeness, we briefly summarize these alternative methods below.

Quantum principal component analysis

Principal component analysis is a dimensionality reduction technique that transforms a large set of correlated variables into a smaller set of uncorrelated variables called principal components. These components capture the directions of maximum variance in the data, allowing one to simplify datasets while preserving most of the original information from high-dimensional patterns.

Classical algorithms for performing PCA scale as $O(d^2)$ in terms of computational complexity and query complexity. In 2014, Lloyd, Mohseni, and Rebentros proposed the quantum principal component analysis (QPCA) algorithm and published their findings in Nature Physics [127]. Here, one uses a quantum random access memory (qRAM) [133] to map a random vector v_j into a quantum state, such that after repeatedly sampling the data, the density matrix for the quantum version of the data actually is the covariance matrix, up to an overall factor. The probing of principal components scales as $\mathcal{O}((\log d)^2)$ in both computational complexity and query complexity, which leads to exponentially more efficient than classical PCA.

Quantum Transformer

Much of the recent AI revolution is due to the transformer architecture, introduced in the "Attention Is All You Need" paper by Google researchers [138]. The classical transformer has obtained many achievements in learning a context-rich representation for a given input sequence, as well as generating new sequences by predicting the next parts of some sequence.

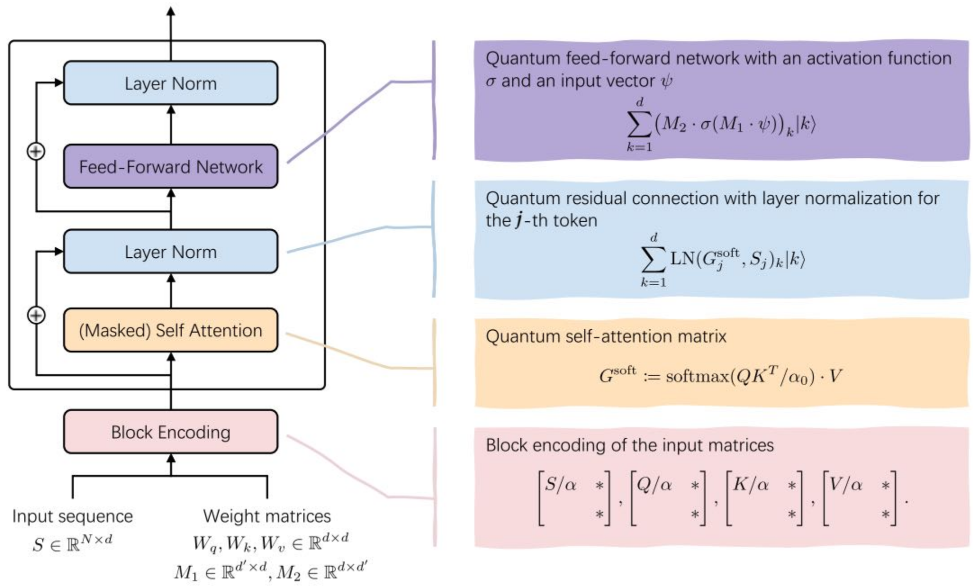


Figure 64: The decoder layer of classical transformer alongside the quantum analogs, adapted from Guo et al. (2024).

Quantum-based attention for transformers, however, are still in their infancy. The motivation for creating a quantum transformer is to reduce the complexity of the self-attention mechanism, which is the bottleneck of the architecture. The traditional classical self-attention mechanism scales $O(n^2 d)$ for sequence length n and embedding dimension d [137]. In this direction of efforts, Quantinuum released an open-source model, Quixer [136], which scales $O(log(nd))$ in the number of qubits and

$O(nlog(d))$ in the number of gates. In contemporary transformer applications, sequence length n is often much larger than the embedding d which makes the logarithmic scaling in the number of qubits with respect to n a promising look into the future of transformers.

Within the industrial application, taking the realm of biochemistry and drug discovery for example, transformer encoders have been developed to extract feature vectors from SMILES strings to be used for downstream predictive tasks, and transformer decoders have been used to generate SMILES strings with prespecified characteristics [142–144].

9.4. Quantum algorithms as subroutine of QML - Fault tolerent era

9.4.1. An introduction to the HHL Algorithm

Recent work has produced quantum algorithms that can serve as building blocks for machine-learning workflows, for example, the HHL algorithm, a *quantum method for solving linear systems*, introduced by Harrow, Hassidim, and Lloyd [126].

HHL in a nutshull

A pioneering quantum routine for solving linear systems of equations of the form $Ax = b$.

Ideals

1. *Quantizes* the problem by expressing the vector $b \in \mathbb{C}^N$ as a quantum $|b\rangle$ over $\log_2 N$ qubits and the vector x as a quantum state $|x\rangle$.
2. *Solve* the equation $Ax = b$ by multiplying both sides by A^{-1}, the HHL algorithm allows one to construct a quantum state proportional to $A^{-1}b$.

Scaling $O[(\log N)^2]$ quantum steps to output $|x\rangle$, compared with the $O(N \log N)$ steps needed to find x using the best known classical method.

Caveats.

- *Information extraction:* The output provides access mainly to certain features of the data, such as moments of x or expectation values $\langle x|B|x\rangle$ for sparse matrices B.
- *Input preparation:* The input vector b must be prepared either on a quantum computer or using qRAM, which may be expensive.
- *Matrix simulation*: The matrix A must be well-conditioned must be possible to simulate e^{-iA} efficiently.

Although HHL scales theoretically as $\mathcal{O}[(\log N)^2]$, realistic resource estimates remain prohibitively large. The current hardware noise and software overhead keep implementations above acceptable error thresholds. Thus, the exponential speedup applies only to restricted classes of linear systems, such that executing HHL at scale remains difficult today. Realizing such algorithms fault-tolerantly is therefore a key milestone for the FTQC era.

9.4.2. HHL: the backbone to many QML tasks

Many machine learning models rely on solving linear equations, a computationally intensive task that often dominates the overall runtime due to the polynomial scaling of complexity with matrix size. The HHL algorithm provides a breakthrough by reducing runtime complexity to poly-logarithmic scaling with matrix size, which is highly significant for ML, where datasets frequently reach sizes in the millions or even billions. Notable examples include quantum principal component analysis [127], quantum support vector machines [128] and quantum-assisted Gaussian process regression [139].

A project to assess HHL-assisted QML for FTQC

In the published work [140], researchers have investigated the potential of HHL-based quantum support vector machine algorithm to the pairwise classification problem. *Pairwise classification* predicts a relation between two entities (e.g., link prediction, chemical interaction). A common classical solver is based on the mapping via the *Kronecker (tensor) kernel*, but solving the resulting systems is costly classically.

Classical Setups. Given training pairs $\{((u_i, v_i), y_i)\}_{i=1}^m$ and the Kronecker kernel over pairs $K_\otimes$, the least-squares SVM (LS-SVM) training solves a linear system

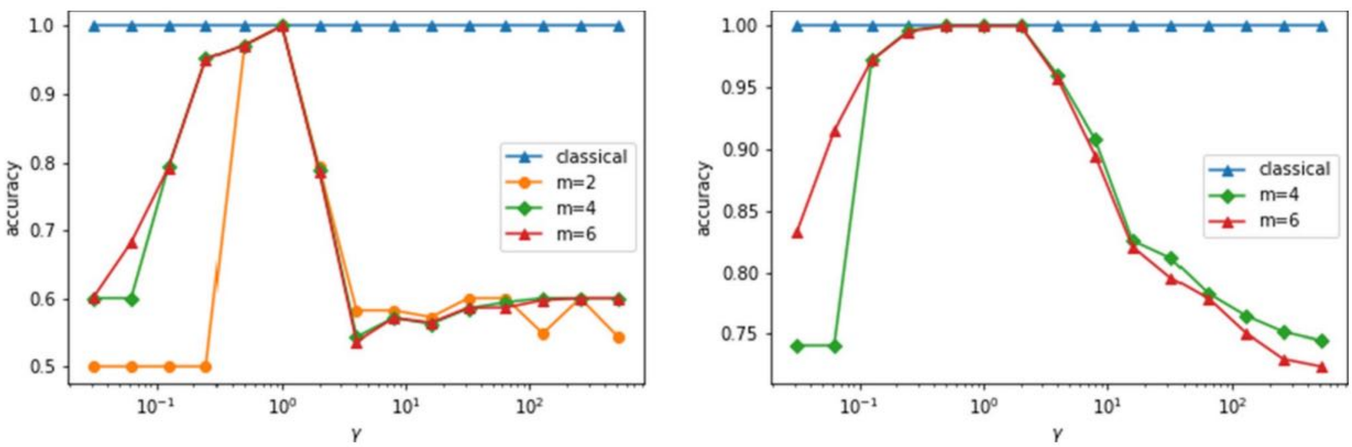
$$\begin{bmatrix} 0 & \mathbf{1}^\top \\ \mathbf{1} & K_\otimes + \lambda I \end{bmatrix} \begin{bmatrix} b \\ \boldsymbol{\alpha} \end{bmatrix} = \begin{bmatrix} 0 \\ \mathbf{y} \end{bmatrix},$$

where $\lambda > 0$ is a regularizer and $\boldsymbol{\alpha}$ are dual weights.

HHL-assisted quantum linear systems solver

1. **State preparation:** Encode the right-hand side (labels) and prepare block-encodings or oracles for K so that $K_\otimes$ can be accessed implicitly via tensor products.
2. **Linear solve:** Use HHL-style quantum linear solvers to obtain a quantum state proportional to $(K_\otimes + \lambda I)^{-1}\mathbf{y}$ (within precision ε).
3. **Prediction:** For a query pair (u, v), estimate $f(u, v) = b + \sum_i \alpha_i K_\otimes\big((u, v), (u_i, v_i)\big)$ via inner-product estimation (swap test or related routines), followed by a sign/threshold decision.

Conclusion



(a) Linear kernel **(b)** Quadratic kernel

Figure 65: The accuracy of the quantum model is sensitive to the value of γ while that of the classical model is insensitive (here larger γ penalizes errors more, fitting the data more tightly)

The QSVM can deliver accuracy comparable to classical algorithms while seems sensitive to γ. The authors supposed that this phenomenon is caused by the limitations on the representation of the eigenvalues in quantum SVM. Eigenvalues are precisely calculated in the classical model, but not necessarily in quantum SVM.

The main purpose of the experiment with small data and the quantum simulator is to confirm implementability. The advantage of the quantum method will become apparent when we apply the method to large data with a real quantum device in the future [140].

9.4.3. Other milestone in FTQC-based QML algorithms

Another milestone for FTQC-era QML is quantum singular value transformation (QSVT), introduced by Gilyén et al. [141]. QSVT performs polynomial transformations of the singular values of a linear operator embedded in a unitary, providing a unifying framework that contains amplitude amplification, quantum linear-system solvers, and quantum simulation. Compared with HHL for linear equations, QSVT achieves better scaling factors, making it a more efficient and versatile tool for QML applications.

9.5. A specific project under EuroHPC JU - LUMI AI Quantum

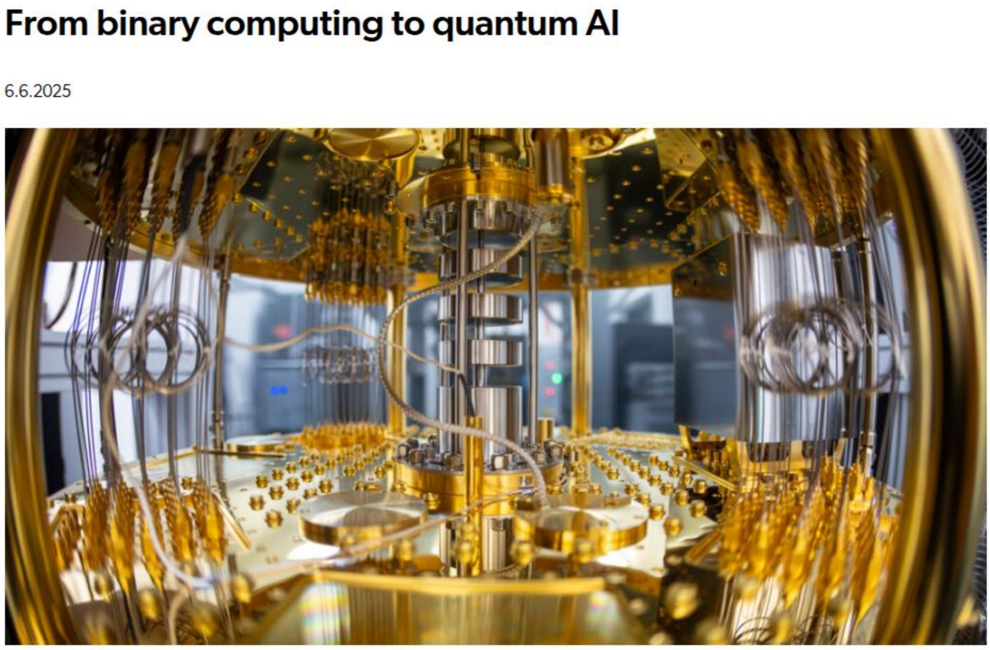


Figure 66: From binary computing to quantum AI. Source: Mikael Johansson Manager of Quantum Technologies CSC – IT Center for Science Ltd, Finland

From a quantum AI perspective, VLQ, the quantum computer of the LUMI-Q consortium located in Ostrava, Czechia, is especially exciting. It is based on a novel “star-shape” layout of the QPU. This design is ideal for creating so-called entangled states. The weirdness of entanglement led Einstein to call it “spooky action at a distance” – in the case of quantum AI, it forms the basis for outperforming purely classical methods.

The happy marriage of quantum and AI is further ensured by the EuroHPC JU AI Factories initiative [145]. Alongside VLQ and other EuroHPC platforms such as EuroQCS-Spain, EuroQCS-Italy and MeluXina-Q, the experimental platform of the LUMI AI Factory [146], LUMI-IQ, will provide a world-leading HPC+AI+QC platform for European end-users in both academia and industry. Through EuroHPC, the entire European RDI community can thus soon get their hands dirty, digging into the promise of quantum-enhanced AI.

Appendix

This appendix collects a range of supplementary materials and references that complement the main body of this report. It provides an overview of relevant *quantum initiatives in Denmark* supported by DeiC quantum department, as well as practical information about *Quantum Backoffice services*, available *funding programs and calls*, and selected *educational activities* such as quantum summer school. Together, these resources aim to support researchers and stakeholders who wish to engage more deeply with the growing Danish quantum ecosystem.

A. Help Desk & Service - DeiC Quantum

As part of Denmark's national quantum strategy, DeiC has received dedicated funding from the government's Research Reserve: DKK 50 million (2023), DKK 40 million (2024), and DKK 40 million (2025). These resources are used to enhance the national digital research infrastructure, with focus on the following initiatives:

- **Q-Competence**: Building national quantum skills and training programs.
- **Q-Algorithm**: Supporting the development of quantum software and algorithms.
- **Q-Access**: Providing user access to quantum computing resources, including through cloud and HPC integration.
- **Q-Net (EuroQCI)**: Participation in the European Quantum Communication Infrastructure initiative, enabling secure quantum networks across Europe.

A.1. Regular Online Office Hour

Currently, DeiC's quantum department holds weekly office hours on **Thursdays from 1:00–2:00 PM (CET)** via Zoom. Please join the meeting using: Zoom link for weekly office hour

A.2. Backoffice - Consulting Service

DeiC's Quantum Back Office portal provides centralized support for quantum computing activities in Denmark. Through this portal, users can:

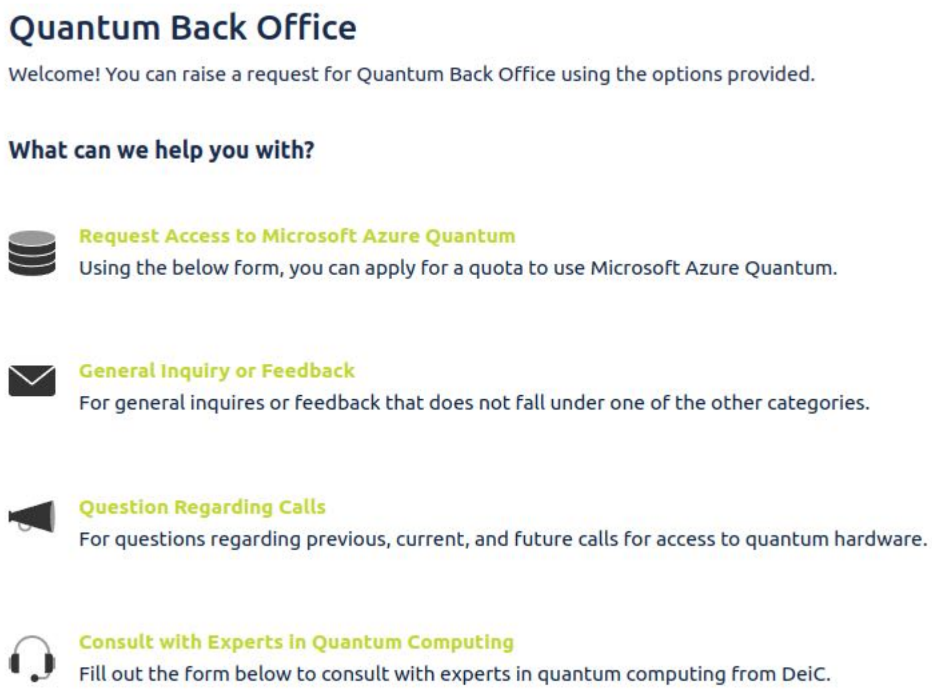


Figure 67: Users can raise a request for Quantum Back Office using the options provided.

A.3. Q-Access Roadshow

The roadshow is designed for both quantum researchers and general audiences interested in learning more about how to access quantum computers and explore potential applications.

In collaboration with the five Danish Quantum Hubs, DeiC invites everyone interested in learning about quantum access—whether you wish to start working with quantum computing yourself, gain additional resources, or simply be informed in order to guide colleagues and collaborators.

Slides from the previous roadshow can be found here: Q-Access Roadshow slides.

Upcoming roadshow

DeiC is also planning to hold the second round of the roadshow in October 2025.

B. Grants and Funding

As a researcher at a Danish university, one has various options to engage in developing the national quantum infrastructure landscape. As part of the implementation of the national quantum strategy, DeiC is facilitating specific areas with programs and funding including development of the next generation of algorithms and software related to future quantum computers and quantum simulator.
E-Grant Application Form Guide
E-Grant Application

All programs and funding are open to researchers in traditional STEM fields and to researchers in other fields such as health sciences, social sciences, and humanities. We also welcome interdisciplinary applications.

All calls are in free and open competition.

Below we list previous calls, which readers can consult for detailed descriptions and requirements:

Quantum Computing Educational Activities

Call opens: March 12, 2025
Call closes: April 30, 2025
Full Call Description
Bid Proposal Template

Postdoc Scholarship on Quantum Algorithms or Quantum Software

Call opens: March 7, 2024
Call closes: April 1, 2025
Full Call Description
Budget Instructions for Postdoc Applications

PhD Scholarship on Quantum Algorithms or Quantum Software

Call opens: November 7, 2024
Call closes: January 7, 2025
Full Call Description

Business PhD Scholarship on Quantum Algorithms or Quantum Software

Call opens: November 7, 2024
Call closes: January 7, 2025
Full Call Description

Business Postdoc Scholarship on Quantum Algorithms or Quantum Software

Call opens: November 7, 2024
Call closes: January 7, 2025
Full Call Description

Access to Quantum Computing Resources

Call opens: November 15, 2024
Call closes: January 6, 2025
Full Call Description
Application Template

For a complete and updated list of opportunities, please visit the DeiC webpage

C. Additional Quantum Initiatives

C.1. Q-Competence

This national competence-building initiative is carried out in alignment with competence-building activities in the EU and related national activities. The initiative aims to disseminate skills and increase understanding of the potential and risks of quantum technology among researchers and businesses. It will target a broad audience and ensure a better understanding of the possibilities and relevance of quantum infrastructure for society.

DeiC will ensure synergy with the activities in the existing EuroCC2 project and engage in collaboration and partnerships with relevant national actors. Therefore, DeiC has created "The National Competence Board for Quantum Computing" (NCB-QC). The board will invite bidders to prepare defined courses, other educational activities and related training material. Find the terms of reference for the Q-Competence working group here.

Members of the working group

- Vice Dean Brian Vinter (AU) — Chairperson
- AAU: Associate Professor Rolf Lyhneborg Lund
- SDU: Professor Jørgen Ellegaard Andersen
- KU: Associate Professor James Emil Avery
- DTU: Senior Researcher Mikkel Heuck
- Alexandra Instituttet: CSO Martin Møller
- DIREC: Director Thomas Riisgaard Hansen
- DeiC: Project Leader Dennis Lange Wollbrink
- DeiC: Quantum Consultant Maria Tammelin Gleerup
- DeiC: Head of Quantum Henrik Navntoft Sønderskov

C.2. Q-Algorithm

The Danish Quantum Algorithm Academy (DQAA) is set up to boost Denmark's work in developing and testing quantum algorithms and the associated software stack. The Academy supports national efforts by offering scholarships for PhD students and postdoctoral researchers, aiming to establish a strong and internationally competitive community around quantum algorithms and related software libraries. In addition, the Academy organises events like workshops, meetings, and guest lectures to foster collaboration and progress in the field, while also building an alumni network to keep past participants engaged.

Working within DeiC's broader quantum programme, the DQA Academy represents the Q-Algorithm branch, partnering with DeiC's Q-Access and Q-Competence initiatives to connect to the broader Danish quantum ecosystem. The Academy Council, with members from each Danish university, oversees the Academy's activities, including the allocation of scholarships and other projects like summer schools, exchange visits, and alumni events. The Council, alongside an expert committee appointed by DeiC, sets criteria for awarding scholarships and evaluates applications.

See Frequently Asked Questions about DQAA scholarships here.

C.3. Q-Net

Denmark participates in the European EuroQCI project, which supports secure data communications based on quantum technology. The Danish project is called QCI.dk and is led by Lecturer Tobias Gehring from DTU Physics. In addition, DTU Photonics, SDU, AAU, KU, the Ministry of Industry, Business, and Financial Affairs, the Ministry of Defence, the Ministry of Foreign Affairs, the Agency for Science and Higher Education, Sparrow Quantum ApS, and of course DeiC are participating. DeiC's role will primarily be to establish and operate the necessary fiber links and possibly also to host some of the repeater stations.

Read more about the EuroQCI project.

Read more about DeiC's participation in the project.

C.4. Niels Bohr Summer School

Quantum Summer School is a new, international initiative and a concrete implementation of the Government's Quantum Strategy from 2023. The goal is to attract top talents within quantum technology to Denmark. The summer school is organized by the Danish e-Infrastructure Consortium (DeiC) in collaboration with Danish universities and Innovation Centre Denmark.

This is a unique opportunity to connect with like-minded peers and industry leaders, expanding your professional network across borders and professional paths.

The summer school starts in Copenhagen in 2025, and in the coming years will move on to the University of Southern Denmark, the Technical University of Denmark and Aalborg University in that order. See the more details of the program organized in 2025 here.

Figure 68: The first round of Niels Bohr Quantum Summer School

About the Programme

Participants dive into topics such as:

- quantum information
- quantum computing theory
- quantum systems
- quantum & AI

Who Can Attend?

Ph.D. students in physics, mathematics, chemistry, computer science, or related fields with foundational knowledge of quantum mechanics who wish to deepen their understanding. A high proficiency in English is required.

Enrollment aims at **35 Danish** and **35 international** students. Among international applicants, priority is given to students from the USA, Israel, South Korea, and Germany (countries highlighted in the Danish National Quantum Strategy and hosting Danish Innovation Centres). Applicants from other countries mentioned in the strategy are considered next, followed by students from the EU or Nordic countries.

Important Dates (2025)

- **10 August:** Arrival day
- **11 August:** Summer school starts at 10:00 (Jagtvej 132)
- **21 August:** QCopenhagen — industry event in connection with the summer school: Q Copenhagen (industry event)
- **22 August:** Summer school ends

References

[1] ’Strategi for Kvanteteknologi: Del 1 - Forskning og innovation i verdensklasse’, Uddannelses- og Forskningsministeriet, 2023. https://ufm.dk/publikationer/2023/strategi-for-kvanteteknologi-del-1-forskning-og-innovation-i-verdensklasse

[2] C.H. Bennett, ‘Logical Reversibility of Computation’, IBM J. Res. Dev. 17 (1973) 525.

[3] R. Landauer, ‘Irreversibility and Heat Generation in the Computing Process’, IBM J. Res. Dev. 5 (1961) 183.

[4] C.H. Bennett, ‘The thermodynamics of computation — a review’, Int J Theor Phys 21, 905–940 (1982).

[5] Paul Benioff, ’Quantum mechanical hamiltonian models of turing machines’, Journal of Statistical Physics, Vol. 29, 515-546, 1982.

[6] A.J.G. Hey and R.W. Allen, eds., ‘The Feynman Lectures on Computation’, (Addison Wesley Longman, Reading MA 1996)

[7] Yuri Manin. ’Computable and Uncomputable’. Sovetskoye Radio, Moscow, 128, 1980

[8] J. Preskill, “Quantum computing 40 years later,” arXiv:2106.10522 [quant-ph].

[9] M. Kjaergaard, M. E. Schwartz, J. Braumüller, P. Krantz, J. I. J. Wang, S. Gustavsson, and W. D. Oliver, “Superconducting Qubits: Current State of Play,” *Annual Review of Condensed Matter Physics*, vol. 11, pp. 369–395, 2020.

[10] T. Strohm *et al.*, “Ion-Based Quantum Computing Hardware: Performance and End-User Perspective,” *arXiv preprint* arXiv:2405.11450 (2024).

[11] L. Henriet *et al.*, “Quantum computing with neutral atoms,” *Quantum* 4, 327 (2020).

[12] T. Lanting *et al.*, “Entanglement in a quantum annealing processor,” *Phys. Rev. X* 4, 021041 (2014).

[13] S. I. Mohtashim *et al.*, “A near-term quantum simulation of the transverse field Ising model hints at Glassy Dynamics,” *Eur. Phys. J. Spec. Top.* (2025).

[14] F. Arute *et al.*, “Quantum supremacy using a programmable superconducting processor,” *Nature* 574, 505–510 (2019).

[15] https://www.ibm.com/think/topics/quantum-computing ’What is quantum computing?’, IBM, 2025.

[16] J. W. Z. Lau, K. H. Lim, H. Shrotriya *et al.*, “NISQ computing: where are we and where do we go?”, *AAPPS Bulletin*, vol. 32, p. 27, 2022.

[17] T. Laakkonen, E. Rinaldi, C. N. Self, E. Chertkov, M. DeCross, D. Hayes, B. Neyenhuis, M. Benedetti and K. Meichanetzidis, “Less Quantum, More Advantage: An End-to-End Quantum Algorithm for the Jones Polynomial,” arXiv:2503.05625 [quant-ph].

[18] Joint Center for Satellite Data Assimilation (JCSDA), "*JEDI Documentation: Overview (Version 1.3.0)*," Available at: https://jointcenterforsatellitedataassimilation-jedi-docs.readthedocs-hosted.com/en/1.3.0/overview/index.html.

[19] M. DeCross, R. Haghshenas, M. Liu, E. Rinaldi, J. Gray, Y. Alexeev, C. H. Baldwin, J. P. Bartolotta, M. Bohn, E. Chertkov, J. Cline, J. Colina, D. Del Vento, J. M. Dreiling, C. Foltz, J. P. Gaebler, T. M. Gatterman, C. N. Gilbreth, J. Giles, D. Gresh, A. Hall, A. Hankin, A. Hansen, N. Hewitt, I. Hoffman, C. Holliman, R. B. Hutson, T. Jacobs, J. Johansen, P. J. Lee, E. Lehman, D. Lucchetti, D. Lykov, I. S. Madjarov, B. Mathewson, K. Mayer, M. Mills, P. Niroula, J. M. Pino, C. Roman, M. Schecter, P. E. Siegfried, B. G. Tiemann, C. Volin, J. Walker, R. Shaydulin, M. Pistoia, S. A. Moses, D. Hayes, B. Neyenhuis, R. P. Stutz, and M. Foss-Feig, "*The Computational Power of Random Quantum Circuits in Arbitrary Geometries*," *arXiv preprint* arXiv:2406.02501, 2024.

[20] P. W. Shor, "Polynomial-Time Algorithms for Prime Factorization and Discrete Logarithms on a Quantum Computer," *SIAM Journal on Scientific and Statistical Computing*, vol. 26, no. 5, pp. 1484–1509, 1997.

[21] S. Ebadi, T. T. Wang, H. Levine, A. Keesling, G. Semeghini, A. Omran, D. Bluvstein, R. Samajdar, H. Pichler, W. W. Ho, S. Choi, S. Sachdev, M. Greiner, V. Vuletić, and M. D. Lukin, "Quantum optimization of maximum independent set using Rydberg atom arrays," *Science*, vol. 376, no. 6598, pp. 1209–1215, 2022.

[22] "16 Danish Quantum Use Cases – December 2024," Danish Quantum Community report, December 2024. Available at: https://dqc.dk/wp-content/uploads/2024/12/16-Danish-Quantum-Use-Cases-December-2024.pdf

[23] https://hpc.llnl.gov/documentation/user-guides/using-el-capitan-systems/hardware-overview 'Using El Capitan Systems: Hardware Overview ', Lawrence Livermore National Laboratory, 2025.

[24] https://quantumatlas.umd.edu/entry/entanglement/ 'Quantum Entanglement', the quantum atlas.

[25] P. B. Upama, M. J. H. Faruk, M. Nazim, M. Masum, H. Shahriar, G. Uddin, S. Barzanjeh, S. I. Ahamed and A. Rahman, "Evolution of Quantum Computing: A Systematic Survey on the Use of Quantum Computing Tools," doi:10.1109/COMPSAC54236.2022.00096 [arXiv:2204.01856 [cs.SE]].

[26] Daley, A.J., Bloch, I., Kokail, C. et al. Practical quantum advantage in quantum simulation. Nature 607, 667–676 (2022). https://doi.org/10.1038/s41586-022-04940-6

[27] 'Choosing hardware for your qsim simulation', Google. https://quantumai.google/qsim/choose

[28] 'Europe takes a quantum leap: LUMI-Q consortium signs contract to establish quantum computer in the Czech Republicn', LUMI. https://www.lumi-supercomputer.eu/europe-takes-a-quantum-leap-lumi-q-consortium-signs-contract/

[29] T. Proctor, K. Young, A. D. Baczewski, and R. Blume-Kohout, "Benchmarking quantum computers," *arXiv preprint* arXiv:2407.08828, 2024.

[30] N. Herrmann, D. Arya, M. W. Doherty, A. Mingare, J. C. Pillay, F. Preis, and S. Prestel, "Quan-

tum utility–definition and assessment of a practical quantum advantage," in *2023 IEEE International Conference on Quantum Software (QSW)*, pp. 162–169, 2023.

[31] D. Lall, A. Agarwal, W. Zhang, L. Lindoy, T. Lindström, S. Webster, S. Hall, N. Chancellor, P. Wallden and R. Garcia-Patron, *et al.* "A Review and Collection of Metrics and Benchmarks for Quantum Computers: definitions, methodologies and software," [arXiv:2502.06717 [quant-ph]].

[32] M. A. Nielsen and I. L. Chuang, *Quantum Computation and Quantum Information*, Cambridge University Press, 2010.

[33] V. V. Shende, S. S. Bullock, and I. L. Markov, "Synthesis of quantum logic circuits," in *Proc. 2005 Asia and South Pacific Design Automation Conference (ASP-DAC)*, pp. [conference pages], 2005.

[34] P. Murali, N. M. Linke, M. Martonosi, A. J. Abhari, N. H. Nguyen, and C. H. Alderete, "Full-stack, real-system quantum computer studies: architectural comparisons and design insights," in *Proceedings of the 46th International Symposium on Computer Architecture (ISCA '19)*, 2019, pp. 5–17.

[35] M. Amy, D. Maslov, M. Mosca, and M. Roetteler, "A meet-in-the-middle algorithm for fast synthesis of depth-optimal quantum circuits," *IEEE Trans. Comput.-Aided Design Integr. Circuits Syst.*, vol. 32, no. 6, pp. 818–830, 2013.

[36] S. J. Devitt, W. J. Munro, and K. Nemoto, "Quantum error correction for beginners," *Reports on Progress in Physics*, vol. 76, no. 7, p. 076001, 2013.

[37] J. Yirka and Y. Subaşı, "Qubit-efficient entanglement spectroscopy using qubit resets," *Quantum*, vol. 5, p. 535, 2021.

[38] R. Yalovetzky, P. Minssen, D. Herman, and M. Pistoia, "Hybrid HHL with dynamic quantum circuits on real hardware," arXiv:2110.15958, 2021.

[39] E. Chertkov, J. Bohnet, D. Francois, J. Gaebler, D. Gresh, A. Hankin, K. Lee, D. Hayes, B. Neyenhuis, R. Stutz, *et al.*, "Holographic dynamics simulations with a trapped-ion quantum computer," *Nature Physics*, vol. 18, pp. 1074–1078, 2022.

[40] E. Chertkov, Z. Cheng, A. C. Potter, S. Gopalakrishnan, T. M. Gatterman, J. A. Gerber, K. Gilmore, D. Gresh, A. Hall, A. Hankin, *et al.*, "Characterizing a non-equilibrium phase transition on a quantum computer," *Nature Physics*, vol. 19, pp. 1799–1805, 2023.

[41] M. Foss-Feig, D. Hayes, J. M. Dreiling, C. Figgatt, J. P. Gaebler, S. A. Moses, J. M. Pino, and A. C. Potter, "Holographic quantum algorithms for simulating correlated spin systems," *Physical Review Research*, vol. 3, p. 033002, 2021.

[42] M. DeCross, E. Chertkov, M. Kohagen, and M. Foss-Feig, "Qubit-Reuse Compilation with Mid-Circuit Measurement and Reset," *Physical Review X*, vol. 13, p. 041057, 2023.

[43] J. Preskill, "Quantum Computing in the NISQ era and beyond," *Quantum*, vol. 2, p. 79, 2018.

[44] I. Siddiqi, "Engineering high-coherence superconducting qubits," *Nature Reviews Materials*, vol. 6, pp. 875–891, 2021.

[45] W. H. Zurek, "Decoherence and the transition from quantum to classical," *Physics Today*, vol. 44, no. 10, pp. 36–44, 1991.

[46] M. Schlosshauer, "Quantum decoherence," *Physics Reports*, vol. 831, pp. 1–57, 2019.

[47] C. H. Bennett and D. P. DiVincenzo, "Quantum information and computation," *Nature*, vol. 404, pp. 247–255, 2000.

[48] D. Bouwmeester and A. Zeilinger, *The Physics of Quantum Information: Quantum Cryptography, Quantum Teleportation, Quantum Computation*, Springer, 2000.

[49] H. E. Brandt, "Qubit devices and the issue of quantum decoherence," *Progress in Quantum Electronics*, vol. 22, pp. 257–370, 1999.

[50] J. Preskill, *Lecture Notes for Physics 229: Quantum Information and Computation*, California Institute of Technology, 1998.

[51] D. Bluvstein, S. J. Evered, A. A. Geim, S. H. Li, H. Zhou, T. Manovitz, S. Ebadi, M. Cain, M. Kalinowski, D. Hangleiter, et al., "Logical quantum processor based on reconfigurable atom arrays," *Nature*, vol. 626, pp. 58–65, 2024.

[52] Y. Kim, A. Eddins, S. Anand, K. X. Wei, E. Van Den Berg, S. Rosenblatt, H. Nayfeh, Y. Wu, M. Zaletel, K. Temme, et al., "Evidence for the utility of quantum computing before fault tolerance," *Nature*, vol. 618, pp. 500–505, 2023.

[53] S. A. Moses, et al., "A Race-Track Trapped-Ion Quantum Processor," *Phys. Rev. X*, vol. 13, 041052, 2023.

[54] N. M. Linke, D. Maslov, M. Roetteler, S. Debnath, C. Figgatt, K. A. Landsman, K. Wright, and C. Monroe, "Experimental Comparison of Two Quantum Computing Architectures," *Proc. Natl. Acad. Sci.*, vol. 114, pp. 3305–3310, 2017.

[55] Google Quantum AI, "Suppressing Quantum Errors by Scaling a Surface Code Logical Qubit," *Nature*, vol. 614, pp. 676–681, 2023.

[56] D. C. McKay, C. J. Wood, S. Sheldon, J. M. Chow, and J. M. Gambetta, "Efficient Z gates for quantum computing," *Phys. Rev. A*, vol. 96, 022330, 2017.

[57] F. Motzoi, J. M. Gambetta, P. Rebentrost, and F. K. Wilhelm, "Simple Pulses for Elimination of Leakage in Weakly Nonlinear Qubits," *Phys. Rev. Lett.*, vol. 103, 110501, 2009.

[58] T. Proctor, S. Seritan, K. Rudinger, E. Nielsen, R. Blume-Kohout, and K. Young, "Scalable Randomized Benchmarking of Quantum Computers Using Mirror Circuits," *Phys. Rev. Lett.* **129**, 150502 (2022).

[59] A. W. Cross, L. S. Bishop, S. Sheldon, P. D. Nation, and J. M. Gambetta, "Validating Quantum Computers Using Randomized Model Circuits," *Phys. Rev. A* **100**, 032328 (2019).

[60] R. Blume-Kohout and K. C. Young, "A Volumetric Framework for Quantum Computer Benchmarks," *Quantum* **4**, 362 (2020).

[61] D. Mills, S. Sivarajah, T. L. Scholten, and R. Duncan, "Application-Motivated, Holistic Benchmarking of a Full Quantum Computing Stack," *Quantum* **5**, 415 (2021).

[62] M. W. Johnson et al., "Quantum Annealing with Manufactured Spins," *Nature* **473**, 194 (2011).

[63] P. Hauke, H. G. Katzgraber, W. Lechner, H. Nishimori, and W. D. Oliver, "Perspectives of quantum annealing: methods and implementations," *Rep. Prog. Phys.* **83**, 054401 (2020).

[64] A. B. Finilla, M. A. Gomez, C. Sebenik, and J. D. Doll, "Quantum Annealing: A New Method for Minimizing Multidimensional Functions," *Chem. Phys. Lett.* **219**, 343 (1994).

[65] C. S. Hamilton, R. Kruse, L. Sansoni, S. Barkhofen, C. Silberhorn, and I. Jex, "Gaussian Boson Sampling," *Phys. Rev. Lett.* **119**, 170501 (2017).

[66] S. Aaronson and A. Arkhipov, "The Computational Complexity of Linear Optics," *Theory Comput.* **9**, 143 (2013).

[67] A. P. Lund, A. Laing, S. Rahimi-Keshari, T. Rudolph, J. L. O'Brien, and T. C. Ralph, "Boson Sampling from a Gaussian State," *Phys. Rev. Lett.* **113**, 100502 (2014).

[68] B. T. Gard, K. R. Motes, J. P. Olson, P. P. Rohde, and J. P. Dowling, "An Introduction to Boson-Sampling," *At. Mesoscale*, 167 (2015).

[69] A. J. Daley, I. Bloch, C. Kokail, S. Flannigan, N. Pearson, M. Troyer, and P. Zoller, "Practical quantum advantage in quantum simulation," *Nature* **607**, 667 (2022).

[70] I. M. Georgescu, S. Ashhab, and F. Nori, "Quantum simulation," *Rev. Mod. Phys.* **86**, 153 (2014).

[71] I. Buluta and F. Nori, "Quantum simulators," *Science* **326**, 108 (2009).

[72] S. Ebadi, T. T. Wang, H. Levine, A. Keesling, G. Semeghini, A. Omran, D. Bluvstein, R. Samajdar, H. Pichler, W. W. Ho, et al., "Quantum phases of matter on a 256-atom programmable quantum simulator," *Nature* **595**, 227 (2021).

[73] M. Kjaergaard, M. E. Schwartz, J. Braumüller, P. Krantz, J. I.-J. Wang, S. Gustavsson, and W. D. Oliver, "Superconducting qubits: Current state of play," *Annu. Rev. Condens. Matter Phys.* **11**, 369 (2020).

[74] P. Krantz, M. Kjaergaard, F. Yan, T. P. Orlando, S. Gustavsson, and W. D. Oliver, "A Quantum Engineer's Guide to Superconducting Qubits," *Appl. Phys. Rev.* **6**, 021318 (2019).

[75] I. Siddiqi, "Engineering high-coherence superconducting qubits," *Nat. Rev. Mater.* **6**, 875 (2021).

[76] M. H. Devoret, A. Wallraff, and J. M. Martinis, "Superconducting qubits: a short review," arXiv:cond-mat/0411174 (2004).

[77] G. Wendin, "Quantum Information Processing with Superconducting Circuits: A Review," *Rep. Prog. Phys.* **80**, 106001 (2017).

[78] J. Clarke and F. K. Wilhelm, "Superconducting quantum bits," *Nature* **453**, 1031 (2008).

[79] M. H. Devoret and R. J. Schoelkopf, "Superconducting circuits for quantum information: an outlook," *Science* **339**, 1169 (2013).

[80] J. You and F. Nori, "Superconducting circuits and quantum information," *Phys. Today* **58**, 42 (2005).

[81] C. D. Bruzewicz, J. Chiaverini, R. McConnell, and J. M. Sage, "Trapped-ion quantum computing: Progress and challenges," *Appl. Phys. Rev.* **6**, 021314 (2019).

[82] H. Häffner, C. F. Roos, and R. Blatt, "Quantum computing with trapped ions," *Phys. Rep.* **469**, 155 (2008).

[83] Z. D. Romaszko, S. Hong, M. Siegele, R. K. Puddy, F. R. Lebrun-Gallagher, S. Weidt, and W. K. Hensinger, "Engineering of microfabricated ion traps and integration of advanced on-chip features," *Nat. Rev. Phys.* **2**, 285 (2020).

[84] J. I. Cirac and P. Zoller, "Quantum Computations with Cold Trapped Ions," *Phys. Rev. Lett.* **74**, 4091 (1995).

[85] J. Benhelm, G. Kirchmair, C. F. Roos, and R. Blatt, "Towards fault-tolerant quantum computing with trapped ions," *Nat. Phys.* **4**, 463 (2008).

[86] C. Monroe and J. Kim, "Scaling the ion trap quantum processor," *Science* **339**, 1164 (2013).

[87] S. Debnath, N. M. Linke, C. Figgatt, K. A. Landsman, K. Wright, and C. Monroe, "Demonstration of a small programmable quantum computer with atomic qubits," *Nature* **536**, 63 (2016).

[88] K. Wright et al., "Benchmarking an 11-Qubit Quantum Computer," *Nat. Commun.* **10**, 5464 (2019).

[89] M. Saffman, "Quantum computing with neutral atoms," *Natl. Sci. Rev.* **6**, 24 (2019).

[90] L. Henriet, L. Beguin, A. Signoles, T. Lahaye, A. Browaeys, G.-O. Reymond, and C. Jurczak, "Quantum computing with neutral atoms," *Quantum* **4**, 327 (2020).

[91] D. Bluvstein et al., "A Quantum Processor Based on Coherent Transport of Entangled Atom Arrays," *Nature* **604**, 451 (2022).

[92] J. Wurtz et al., "Aquila: QuEra's 256-qubit neutral-atom quantum computer," arXiv:2306.11727 (2023).

[93] K. Wintersperger et al., "Neutral atom quantum computing hardware: performance and end-user perspective," *EPJ Quantum Technol.* **10**, 32 (2023).

[94] T. M. Graham et al., "Multi-Qubit Entanglement and Algorithms on a Neutral-Atom Quantum Computer," *Nature* **604**, 457 (2022).

[95] D. Bluvstein et al., "Logical quantum processor based on reconfigurable atom arrays," *Nature* **626**, 58 (2024).

[96] J. L. O'Brien, A. Furusawa, and J. Vučković, "Photonic quantum technologies," *Nat. Photonics* **3**, 687 (2009).

[97] P. Kok et al., "Linear optical quantum computing with photonic qubits," *Rev. Mod. Phys.* **79**, 135 (2007).

[98] J. Wang, F. Sciarrino, A. Laing, and M. G. Thompson, "Integrated photonic quantum technologies," *Nat. Photonics* **14**, 273 (2020).

[99] S. Barz, “Quantum computing with photons: introduction to the circuit model, the one-way quantum computer, and the fundamental principles of photonic experiments,” *J. Phys. B: At. Mol. Opt. Phys.* **48**, 083001 (2015).

[100] F. Flamini, N. Spagnolo, and F. Sciarrino, “Photonic quantum information processing: a review,” *Rep. Prog. Phys.* **82**, 016001 (2018).

[101] S. Slussarenko and G. J. Pryde, “Photonic quantum information processing: a concise review,” *Appl. Phys. Rev.* **6**, 041303 (2019).

[102] E. Knill, R. Laflamme, and G. J. Milburn, “A scheme for efficient quantum computation with linear optics,” *Nature* **409**, 46 (2001).

[103] G. Burkard et al., “Semiconductor spin qubits,” *Rev. Mod. Phys.* **95**, 025003 (2023).

[104] A. Chatterjee et al., “Semiconductor qubits in practice,” *Nat. Rev. Phys.* **3**, 157 (2021).

[105] M. D. Michielis et al., “Silicon spin qubits from laboratory to industry,” *J. Phys. D: Appl. Phys.* **56**, 363001 (2023).

[106] R. Maurand et al., “A CMOS silicon spin qubit,” *Nat. Commun.* **7**, 13575 (2016).

[107] D. D. Awschalom, L. C. Bassett, A. S. Dzurak, E. L. Hu, and J. R. Petta, “Quantum spintronics: engineering and manipulating atom-like spins in semiconductors,” *Science* **339**, 1174 (2013).

[108] L. M. Vandersypen and M. A. Eriksson, “Quantum computing with semiconductor spins,” *Phys. Today* **72**, 38 (2019).

[109] C. Kloeffel and D. Loss, “Prospects for spin-based quantum computing in quantum dots,” *Annu. Rev. Condens. Matter Phys.* **4**, 51 (2013).

[110] D. J. Wineland, C. Monroe, W. M. Itano, D. Leibfried, B. E. King, and D. M. Meekhof, “Experimental issues in coherent quantum-state manipulation of trapped atomic ions,” *J. Res. Natl. Inst. Stand. Tech.* **103**, 259 (1998).

[111] K. R. Brown, J. Chiaverini, J. M. Sage, and H. Häffner, “Materials challenges for trapped-ion quantum computers,” *Nature Reviews Materials* **6**(10), 892–905 (2021).

[112] M. Endres, H. Bernien, A. Keesling, H. Levine, E. R. Anschuetz, A. Krajenbrink, C. Senko, V. Vuletić, M. Greiner, and M. D. Lukin, “Atom-by-atom assembly of defect-free one-dimensional cold atom arrays,” *Science* **354**, 1024 (2016).

[113] D. Barredo, S. de Léséleuc, V. Lienhard, T. Lahaye, and A. Browaeys, “An atom-by-atom assembler of defect-free arbitrary two-dimensional atomic arrays,” *Science* **354**, 1021 (2016).

[114] D. Barredo, V. Lienhard, S. de Léséleuc, T. Lahaye, and A. Browaeys, “Synthetic Three-Dimensional Atomic Structures Assembled Atom by Atom,” *Nature* **561**, 79 (2018).

[115] P. Huft, Y. Song, T. M. Graham, K. Jooya, S. Deshpande, C. Fang, M. Kats, and M. Saffman, “Simple, passive design for large optical trap arrays for single atoms,” *Phys. Rev. A* **105**, 063111 (2022).

[116] K. Barnes et al., “Assembly and Coherent Control of a Register of Nuclear Spin Qubits,” *Nat. Commun.* **13**, 2779 (2022).

[117] B. Nikolov, E. Diamond-Hitchcock, J. Bass, N. L. R. Spong, and J. D. Pritchard, “Randomized Benchmarking Using Nondestructive Readout in a Two-Dimensional Atom Array,” *Phys. Rev. Lett.* **131**, 030602 (2023).

[118] D. Jaksch, J. I. Cirac, P. Zoller, S. L. Rolston, R. Côté, and M. D. Lukin, “Fast Quantum Gates for Neutral Atoms,” *Phys. Rev. Lett.* **85**, 2208 (2000).

[119] M. D. Lukin, M. Fleischhauer, R. Côté, L. M. Duan, D. Jaksch, J. I. Cirac, and P. Zoller, “Dipole Blockade and Quantum Information Processing in Mesoscopic Atomic Ensembles,” *Phys. Rev. Lett.* **87**, 037901 (2001).

[120] X.-F. Shi, “Quantum logic and entanglement by neutral Rydberg atoms: methods and fidelity,” *Quantum Sci. Technol.* **7**, 023002 (2022).

[121] L. S. Madsen, F. Laudenbach, M. Falconi, J. B. Bulmer, F. H. Fratini, M. M. Valerius, C. B. Mikkelsen, J. H. Appel, J. P. Hansen, J. L. Nielsen, et al., “Quantum Computational Advantage with a Programmable Photonic Processor,” Nature **606**, 75 (2022).

[122] S. Shi, B. Xu, K. Zhang, G. S. Ye, D. S. Xiang, Y. Liu, J. Wang, D. Su, and L. Li, “High-fidelity photonic quantum logic gate based on near-optimal Rydberg single-photon source,” Nat. Commun. **13**, 4454 (2022).

[123] M. Hibat-Allah, M. Mauri, J. Carrasquilla, and A. Perdomo-Ortiz, “A framework for demonstrating practical quantum advantage: comparing quantum against classical generative models,” Commun. Phys. **7**, 68 (2024).

[124] Maria Schuld and Francesco Petruccione, *Machine Learning with Quantum Computers*, 2nd ed., Quantum Science and Technology series, Springer, Cham, 2021. ISBN: 978-3-030-83098-4.

[125] K. Zaman, T. Ahmed, M. A. Hanif, A. Marchisio, and M. Shafique, “A Comparative Analysis of Hybrid-Quantum Classical Neural Networks,” In *Grid, Cloud, and Cluster Computing; Quantum Technologies; and Modeling, Simulation and Visualization Methods — Revised Selected Papers*, edited by H. R. Arabnia, M. Takata, L. Deligiannidis, P. Rivas, M. Ohue, and N. Yasuo, *Communications in Computer and Information Science*, vol. 2257 CCIS, pp. 102–115, Springer, 2025.

[126] A. W. Harrow, A. Hassidim, and S. Lloyd, “Quantum Algorithm for Linear Systems of Equations,” arXiv:0811.3171 [quant-ph] (version 3, 30 September 2009).

[127] S. Lloyd, M. Mohseni, and P. Rebentrost, “Quantum principal component analysis,” Nature Phys. **10**, 631–633 (2014). doi:10.1038/nphys3029

[128] P. Rebentrost, M. Mohseni, and S. Lloyd, “Quantum support vector machine for big data classification,” Phys. Rev. Lett. **113**, 130503 (2014). doi:10.1103/PhysRevLett.113.130503

[129] J. Biamonte, P. Wittek, N. Pancotti, P. Rebentrost, N. Wiebe, and S. Lloyd, “Quantum Machine Learning,” *Nature*, vol. 549, pp. 195–202, 2017.

[130] T. Laakkonen, E. Rinaldi, C. N. Self, E. Chertkov, M. DeCross, D. Hayes, B. Neyenhuis, M. Benedetti, and K. Meichanetzidis, “Less Quantum, More Advantage: An End-to-End Quantum Algorithm for the Jones Polynomial,” arXiv preprint arXiv:2503.05625, 2025.

[131] P. Sen, A. S. Bhatia, K. S. Bhangu, and A. Elbeltagi, “Variational Quantum Classifiers Through the Lens of the Hessian,” arXiv preprint arXiv:2105.10162, 2021.

[132] A. Ajagekar and F. You, “Molecular design with automated quantum computing-based deep learning and optimization,” npj Comput. Mater., vol. 9, Art. no. 143, 2023.

[133] V. Giovannetti, S. Lloyd, and L. Maccone, “Quantum random access memory,” *Physical Review Letters*, vol. 100, no. 16, p. 160501, 2008.

[134] V. N. Vapnik, *The Nature of Statistical Learning Theory*, Springer, New York, 1995.

[135] Z. Li, X. Liu, N. Xu, and J. Du, “Experimental realization of a quantum support vector machine,” *Physical Review Letters*, vol. 114, no. 14, p. 140504, 2015.

[136] N. Khatri, G. Matos, L. Coopmans, and S. Clark, “Quixer: A Quantum Transformer Model,” *Preprint* at `http://arxiv.org/abs/2406.04305`, arXiv:2406.04305 [quant-ph], 2024.

[137] A. M. Smaldone, Y. Shee, G. W. Kyro, C. Xu, N. P. Vu, R. Dutta, M. H. Farag, A. Galda, S. Kumar, E. Kyoseva, and V. S. Batista, ”Quantum Machine Learning in Drug Discovery: Applications in Academia and Pharmaceutical Industries,” preprint arXiv:2409.15645, Sep. 2024.

[138] A. Vaswani, N. Shazeer, N. Parmar, J. Uszkoreit, L. Jones, A. N. Gomez, L. Kaiser, and I. Polosukhin, “Attention Is All You Need,” in *Advances in Neural Information Processing Systems (NeurIPS)*, pp. 5998–6008, 2017.

[139] Z. Zhao, J. K. Fitzsimons, and J. F. Fitzsimons, “Quantum-assisted Gaussian process regression,” *Phys. Rev. A* **99**, 052331 (2019). doi:10.1103/PhysRevA.99.052331

[140] T. Nohara, S. Oyama, and I. Noda, “Pairwise classification using quantum support vector machine with Kronecker kernel,” *Quantum Machine Intelligence*, vol. 4, no. 2, Article 22, 2022. DOI: `10.1007/s42484-022-00082-0`.

[141] A. Gilyèn, Y. Su, G. H. Low, and N. Wiebe, “Quantum singular value transformation and beyond: exponential improvements for quantum matrix arithmetics,” in *Proc. 51st ACM SIGACT Symposium on Theory of Computing (STOC 2019)*, pp. 193–204, 2019. doi:10.1145/3313276.3316366.

[142] B. Ramsundar, P. Eastman, P. Gomes, K. Leswing, and Z. Pande, *Deep Learning for the Life Sciences: Applying Deep Learning to Genomics, Microscopy, Drug Discovery, and More*. O’Reilly Media, 2019.

[143] P. Schwaller, T. Laino, T. Gaildrat, O. Bekas, and A. A. Lee, “Molecular Transformer: A Model for Uncertainty-Calibrated Chemical Reaction Prediction,” *ACS Central Science*, vol. 5, no. 9, pp. 1572–1583, 2019.

[144] S. Honda, S. Shi, and H. Ueda, “SMILES Transformer: Pre-trained Molecular Fingerprint for Low Data Drug Discovery,” *arXiv preprint arXiv:1911.04738*, 2019.

[145] EuroHPC Joint Undertaking, *Selection of the First Seven AI Factories to Drive Europe’s Leadership in AI* (press release), 10 Dec 2024. Available at: `https://eurohpc-ju.europa.eu/`

selection-first-seven-ai-factories-drive-europes-leadership-ai-2024-12-en.

[146] LUMI Consortium, *LUMI AI Factory*. Available at: https://www.lumi-supercomputer.eu/lumi-ai-factory/.